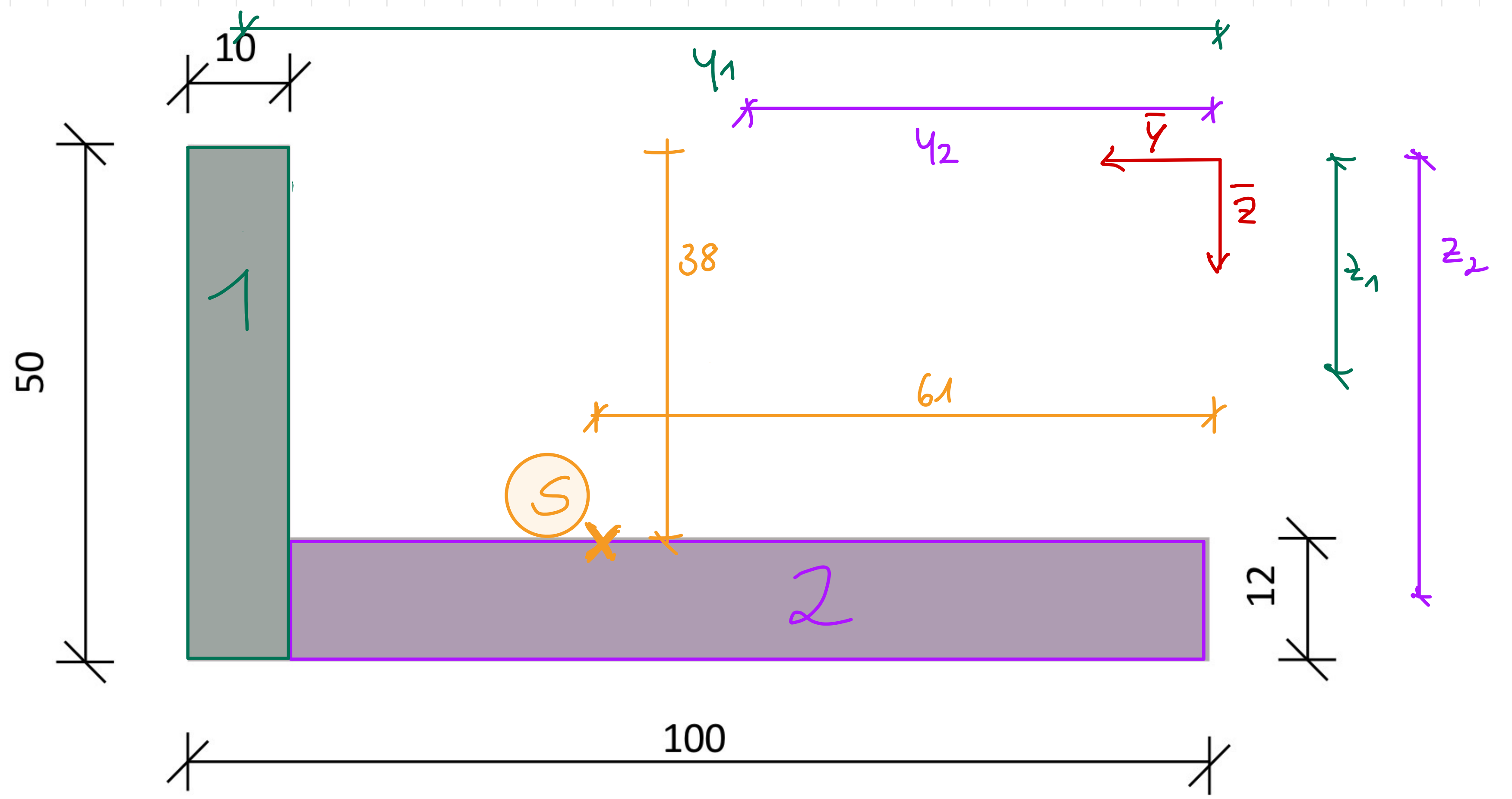
Übung 7

Aufgabe 1



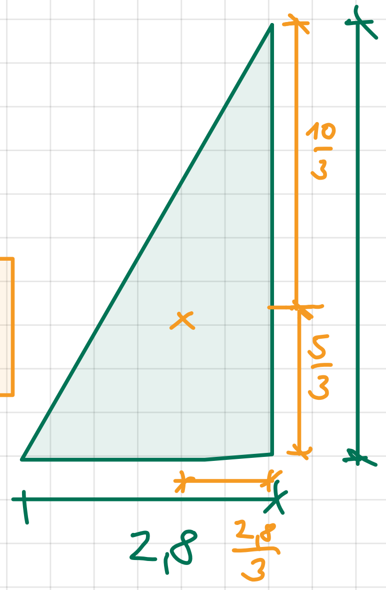
$$\bar{y}_S = \frac{\sum_{i=1}^{n} A_i \cdot \bar{y}_i}{\sum_{i=1}^{n} A_i}$$

$$\bar{z}_S = \frac{\sum_{i=1}^{n} A_i \cdot \bar{z}_i}{\sum_{i=1}^{n} A_i}$$

$$\bar{y}_s = \frac{\overbrace{5 \cdot 1}^{A_1} \cdot \overbrace{\left(10 - \frac{1}{2}\right)}^{\bar{y}_1} + \overbrace{9 \cdot 1{,}2}^{A_2} \cdot \overbrace{\frac{9}{2}}^{\bar{y}_2}}{\underbrace{5 \cdot 1}_{A_1} + \underbrace{9 \cdot 1{,}2}_{A_2}} \approx 6{,}08\,\text{cm}$$

$$\bar{z}_S = \frac{\overbrace{5 \cdot 1}^{A_1} \cdot \overbrace{\frac{5}{2}}^{y_1} + \overbrace{9 \cdot 1{,}2}^{A_2} \cdot \overbrace{\left(5 - \frac{1{,}2}{2}\right)}^{y_2}}{\underbrace{5 \cdot 1}_{A_1} + \underbrace{9 \cdot 1{,}2}_{A_2}} \approx 3{,}80\,\text{cm}$$

Aufgabe 2

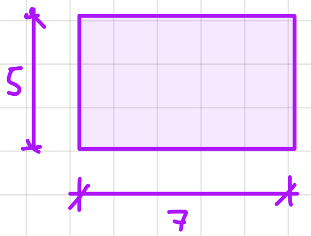


$$A_1 = 5 \cdot 2{,}8 \cdot \frac{1}{2} = 7\,\text{cm}^2$$

Schwerpunkt bei $\frac{1}{3}$ der Seitenlänge

$$\bar{y}_1 = 2{,}4 + 2{,}5 + \frac{2{,}8}{3} = \frac{31}{3}\,\text{cm}$$

$$\bar{z}_1 = 5 - \frac{5}{3} = \frac{10}{3}\,\text{cm}$$

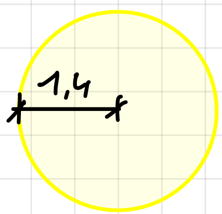


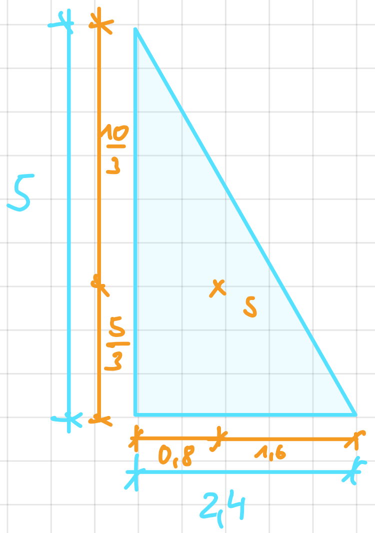
$$A_2 = (5+2) \cdot (1{,}8 + 3{,}2) = 35\,\text{cm}^2$$

$$\bar{y}_2 = 2{,}4 + \frac{7}{2} = 5{,}9\,\text{cm} \;;\; \bar{z}_2 = \frac{5}{2} = 2{,}5\,\text{cm}$$

$$A_3 = \pi \cdot r^2 = \pi \cdot 1{,}4^2 = \frac{49}{25}\pi\,\text{cm}^2$$

$$\bar{y}_3 = 2{,}4 + 2 = 4{,}4\,\text{cm} \;;\; \bar{z}_3 = 3{,}2\,\text{cm}$$

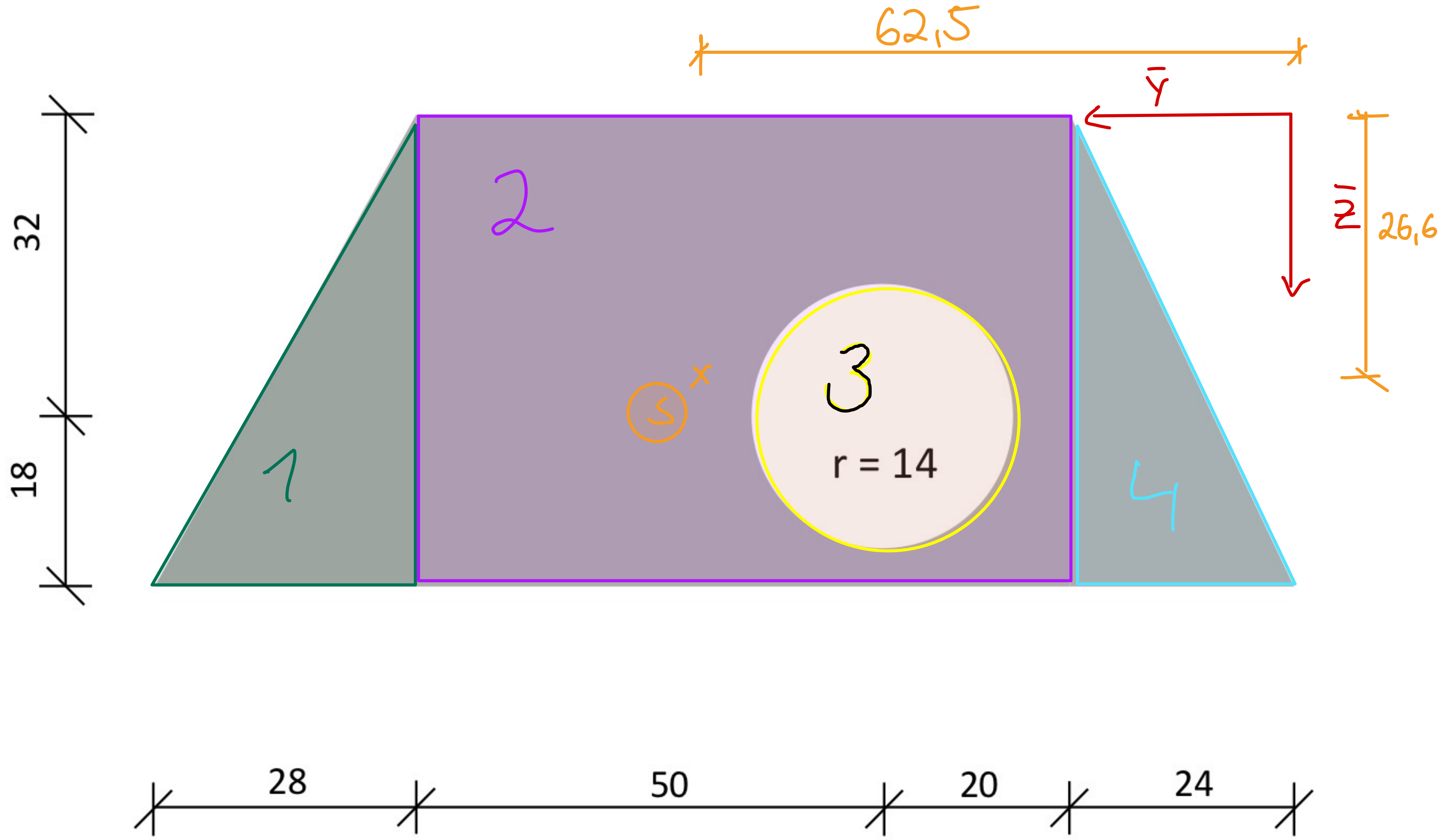




$$A_4 = 5 \cdot 2{,}4 \cdot \frac{1}{2} = 6\,\text{cm}^2$$

$$\bar{y}_4 = 1{,}6\,\text{cm}$$

$$\bar{z}_4 = \frac{10}{3}\,\text{cm}$$



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$$\bar{y}_S = \frac{7 \cdot \frac{31}{3} + 35 \cdot 5{,}9 - \frac{49}{25} \cdot \pi \cdot 4{,}4 + 6 \cdot 1{,}6}{7 + 35 - \frac{49}{25} \cdot \pi + 6} \approx 6{,}25\,\text{cm}$$

$$\bar{z}_S = \frac{7 \cdot \frac{10}{3} + 35 \cdot 2{,}5 - \frac{49}{25} \cdot \pi \cdot 3{,}2 + 6 \cdot \frac{10}{3}}{7 + 35 - \frac{49}{25}\pi + 6} \approx 2{,}66\,\text{cm}$$

Aufgabe 3

Schwerpunkt:

$\bar{y}_s = 0$ (wegen Symmetrie)

$$\bar{z}_s = \frac{\overbrace{10\cdot 1{,}2\cdot\frac{1{,}2}{2}}^{1} + \overbrace{9{,}5\cdot 1\cdot\left(1{,}2+\frac{9{,}5}{2}\right)}^{2} + \overbrace{7{,}5\cdot 0{,}9\cdot\left(1{,}2+9{,}5+\frac{0{,}9}{2}\right)}^{3}}{\underbrace{10\cdot 1{,}2}_{1} + \underbrace{9{,}5\cdot 1}_{2} + \underbrace{7{,}5\cdot 0{,}9}_{3}} \approx 4{,}92\,\text{cm}$$

Flächenträgheitsmomente:

$$I_y = \sum I_{y_i} + \sum z_i^2\cdot A_i \qquad I_z = \sum I_{z_i} + \sum y_i^2\cdot A_i$$

$$I_{y_i} = \frac{b\cdot h^3}{12} \qquad I_{z_i} = \frac{h\cdot b^3}{12}$$

$$I_y = \overbrace{\frac{10\cdot 1{,}2^3}{12} + \left(4{,}92-\frac{1{,}2}{2}\right)^2\cdot 10\cdot 1{,}2}^{1} = 225{,}39$$

$$+ \overbrace{\frac{1\cdot 9{,}5^3}{12} + \left(4{,}92-\left(1{,}2+\frac{9{,}5}{2}\right)\right)^2\cdot 1\cdot 9{,}5}^{2} = 81{,}53$$

$$+ \underbrace{\frac{7{,}5\cdot 0{,}9^3}{12} + \left(4{,}92-\left(1{,}2+9{,}5+\frac{0{,}9}{2}\right)\right)^2\cdot 0{,}9\cdot 7{,}5}_{3} = 262{,}44$$

$$= 569{,}36\ \text{cm}^4$$

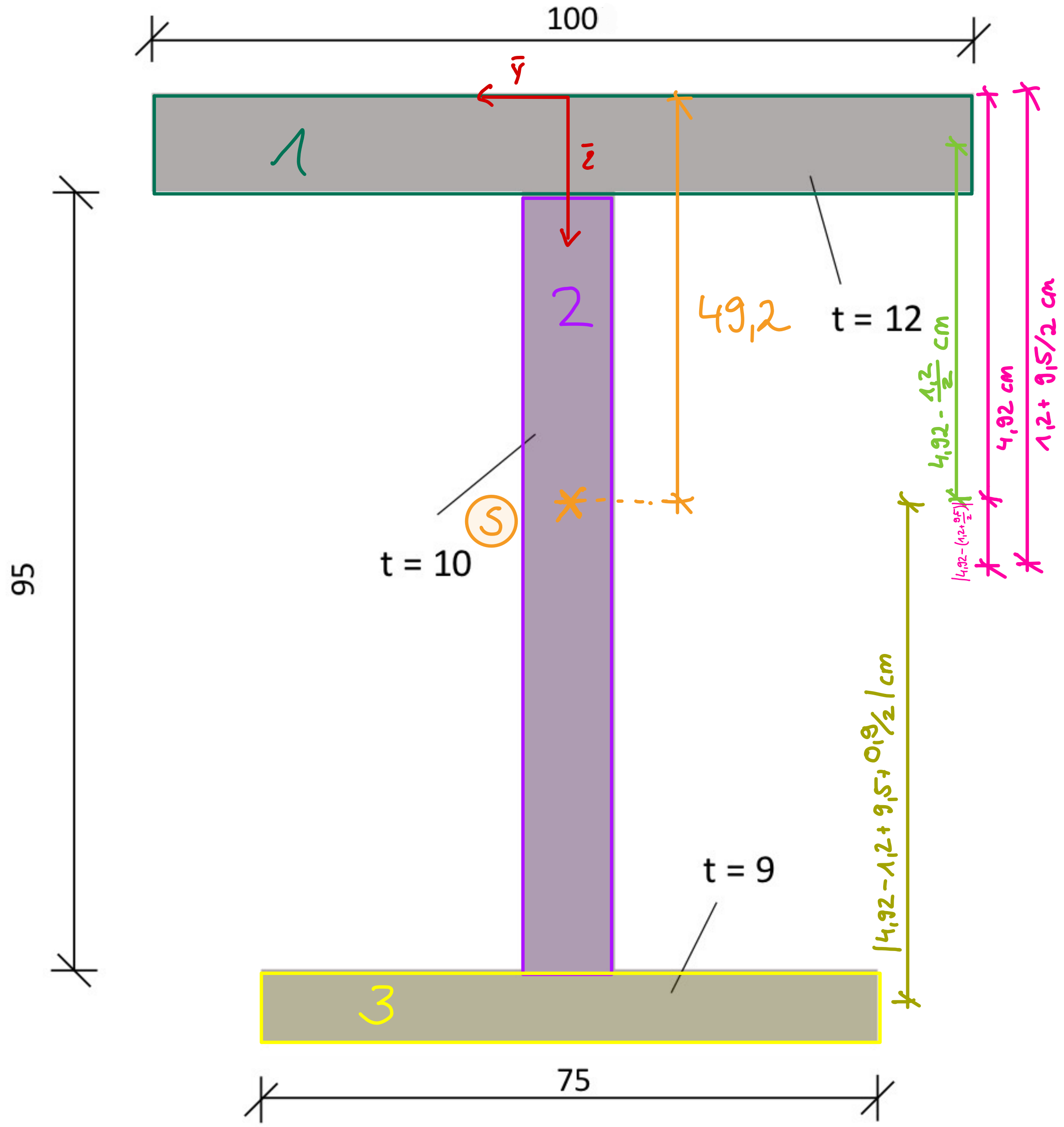
$$I_z = \overbrace{\frac{1{,}2\cdot 10^3}{12}}^{1} = 100$$

$$+ \overbrace{\frac{9{,}5\cdot 1^3}{12}}^{2} = 0{,}77$$

$$+ \overbrace{\frac{0{,}9\cdot 7{,}5^3}{12}}^{3} = 31{,}64$$

$\leftarrow$ hier entfällt $\sum y_i^2\cdot A_i$, weil $y_i = 0$ (bei allen Teilen)

$$= 132{,}41\ \text{cm}^4$$

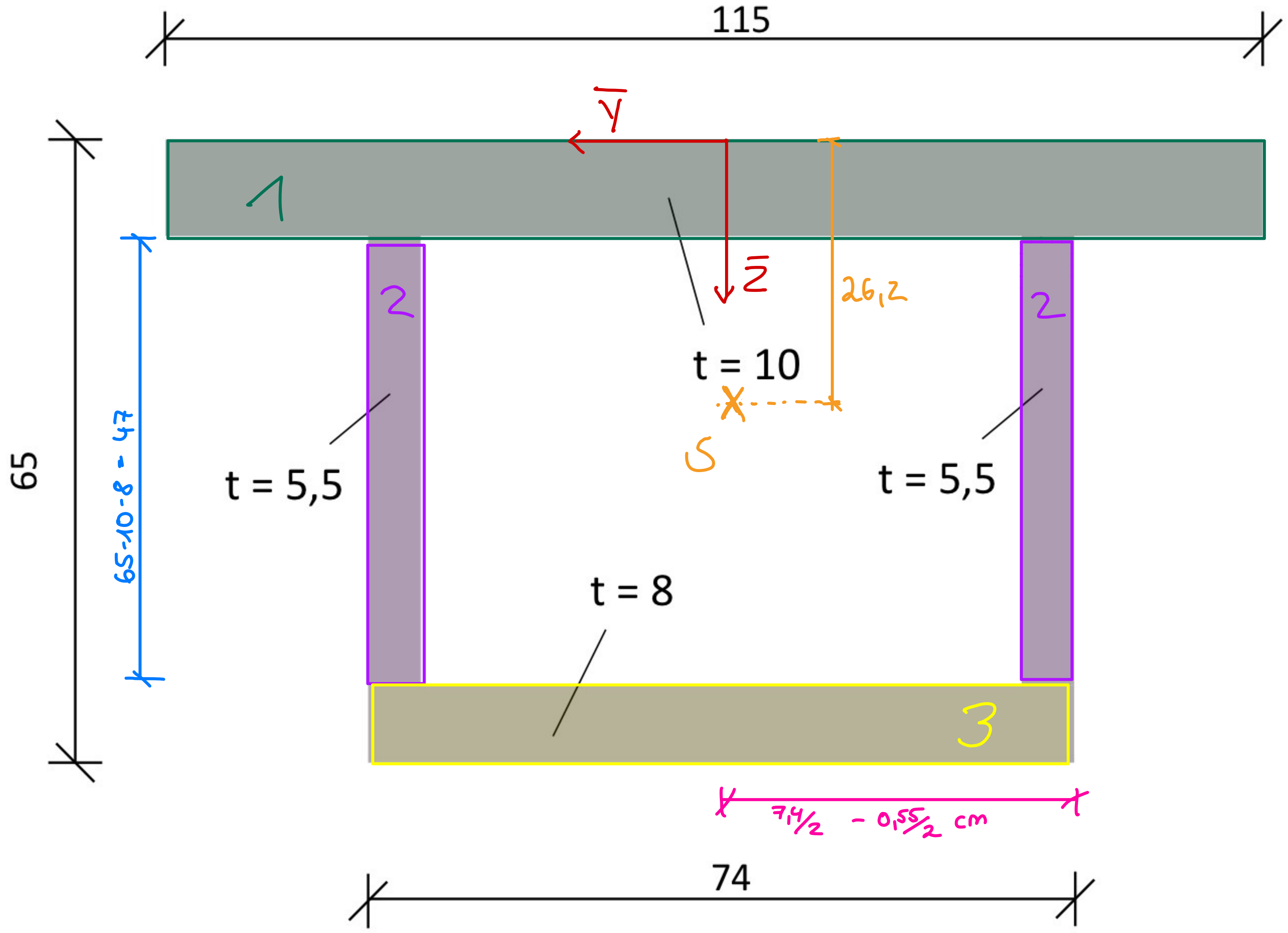


Aufgabe 4

$\bar{y}_s = 0$ (Symmetrie)

$$\bar{z}_s = \frac{\overbrace{11{,}5\cdot 1\cdot\frac{1}{2}}^{1} + \overbrace{2\cdot\left(4{,}7\cdot 0{,}55\cdot\left(1+\frac{4{,}7}{2}\right)\right)}^{2\cdot 2} + \overbrace{0{,}8\cdot 7{,}4\cdot\left(6{,}5-\frac{0{,}8}{2}\right)}^{3}}{\underbrace{11{,}5\cdot 1}_{1} + \underbrace{2\cdot 4{,}7\cdot 0{,}55}_{2\cdot 2} + \underbrace{7{,}4\cdot 0{,}8}_{3}}$$

$\approx 2{,}62\,\text{cm}$



Flächenträgheitsmomente

$I_y = \sum I_{yi} + \sum z_i^2 \cdot A_i$

$I_z = \sum I_{zi} + \sum y_i^2 \cdot A_i$

$$I_y = \overbrace{\frac{11{,}5\cdot 1^3}{12} + \left(2{,}62-\frac{1}{2}\right)^2\cdot 1\cdot 11{,}5}^{1} = 52{,}64$$

$$+ \overbrace{2\cdot\left(\frac{0{,}55\cdot 4{,}7^3}{12} + \left(2{,}62-\left(1+\frac{4{,}7}{2}\right)\right)^2\cdot 0{,}55\cdot 4{,}7\right)}^{2\cdot 2} = 12{,}27$$

$$+ \underbrace{\frac{7{,}4\cdot 0{,}8^3}{12} + \left(2{,}62-\left(1+4{,}7+\frac{0{,}8}{2}\right)\right)^2\cdot 0{,}8\cdot 7{,}4}_{3} = 72{,}0$$

$$= 136{,}91\,\text{cm}^4$$

$$I_z = \frac{1\cdot 11{,}5^3}{12} = 126{,}74$$

$$+ 2\cdot\left(\frac{4{,}7\cdot 0{,}55^3}{12} + \left(\frac{7{,}4}{2}-\frac{0{,}55}{2}\right)^2\cdot 0{,}55\cdot 4{,}7\right) = 60{,}78$$

$$+ \frac{0{,}8\cdot 7{,}4^3}{12} = 27{,}01$$

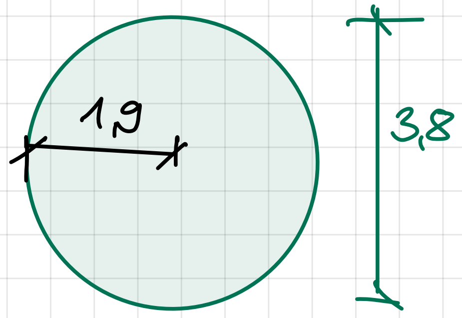
$$= 214{,}53\,\text{cm}^4$$

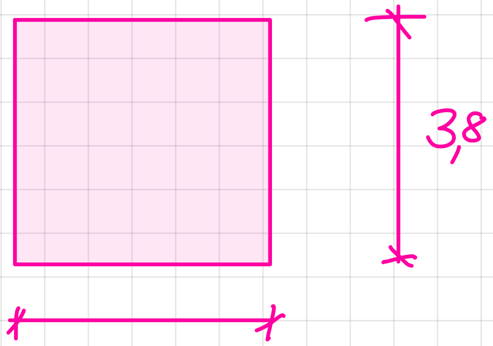
Aufgabe 5

$\bar{y}_s = 0$ (Symmetrie)

$A_1 = \pi \cdot 1{,}9^2 = \frac{361}{100}\pi \text{ cm}^2$ $(\approx 11{,}34 \text{ cm}^2)$

$\bar{z}_1 = 1{,}9 \text{ cm}$





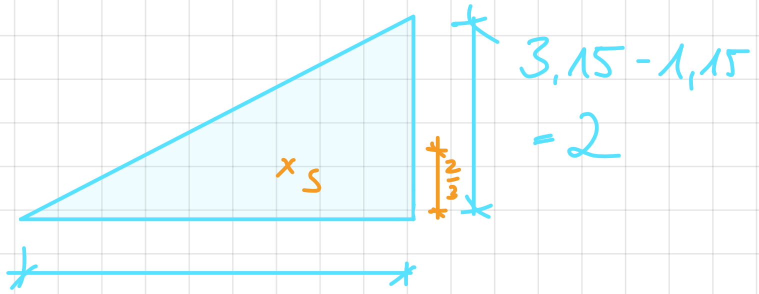
$72 - 1{,}9 \cdot 2$
$= 3{,}4$

$A_2 = 3{,}4 \cdot 3{,}8 = 12{,}92 \text{ cm}^2$

$\bar{z}_2 = \frac{38}{2} = 1{,}9 \text{ cm}$

$A_3 = (17{,}2 - 3{,}8 - 1{,}15) \cdot 1{,}65 = 20{,}2 \text{ cm}^2$

$\bar{z}_3 = 3{,}8 + \frac{17{,}2 - 3{,}8 - 1{,}15}{2} = 9{,}93 \text{ cm}$



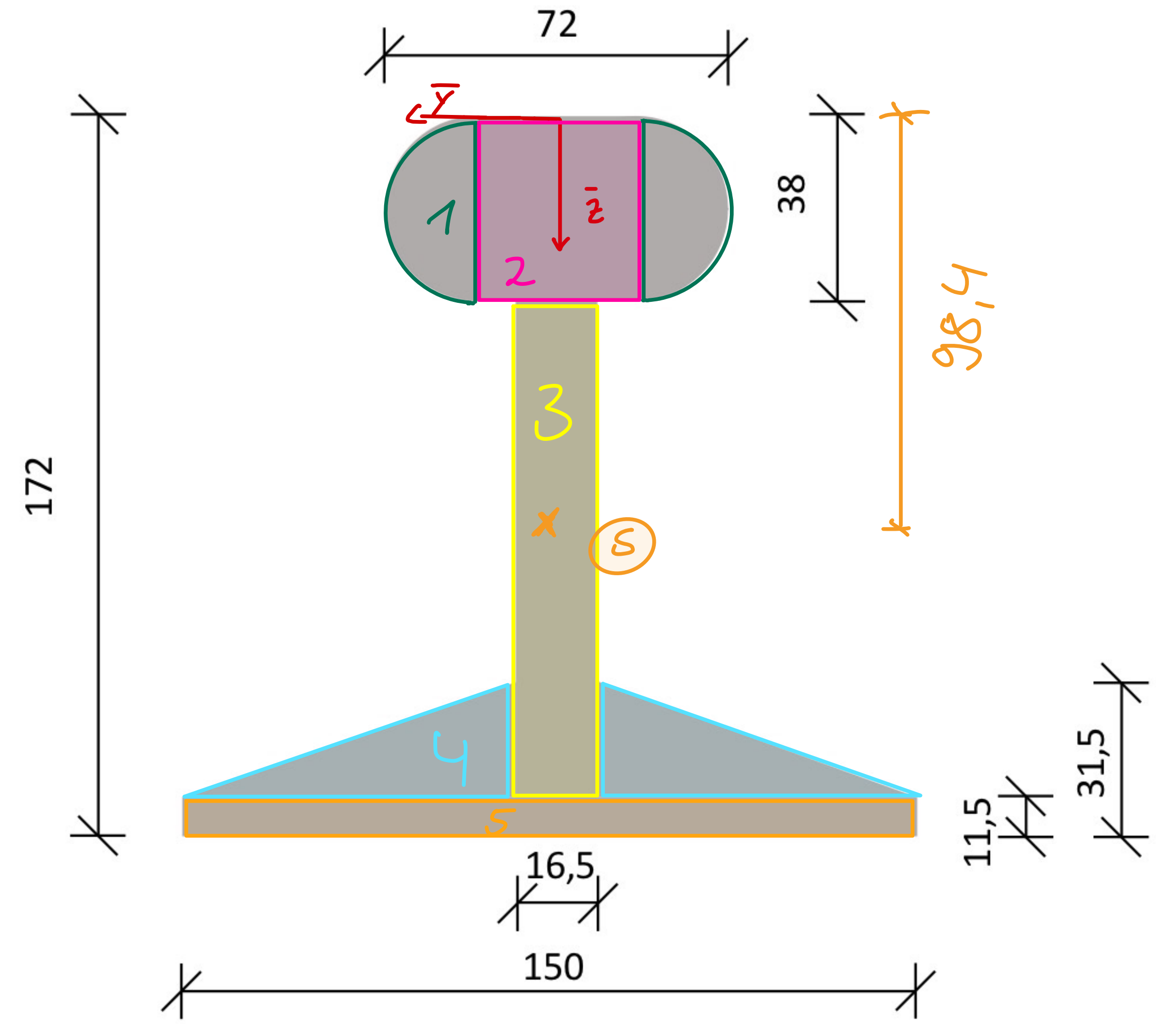
$\frac{15 - 1{,}65}{2} = 6{,}68 \text{ cm}$

$A_4 = \frac{1}{2} \cdot 2 \cdot 6{,}68 = 6{,}68 \text{ cm}^2$

$z_4 = 17{,}2 - 1{,}15 - \frac{2}{3} = 15{,}38 \text{ cm}$

$A_5 = 15 \cdot 1{,}15 = 17{,}25 \text{ cm}^2$

$\bar{z}_5 = 17{,}2 - \frac{1{,}15}{2} = 16{,}63 \text{ cm}$



$$\bar{z}_S = \frac{\frac{361}{100}\cdot\pi \cdot 1{,}9 + 12{,}92 \cdot 1{,}9 + 20{,}2 \cdot 9{,}93 + 2 \cdot 6{,}68 \cdot 15{,}38 + 17{,}25 \cdot 16{,}63}{\frac{361}{100}\pi + 12{,}92 + 20{,}2 + 2 \cdot 6{,}68 + 17{,}25}$$

$\approx 9{,}84 \text{ cm}$