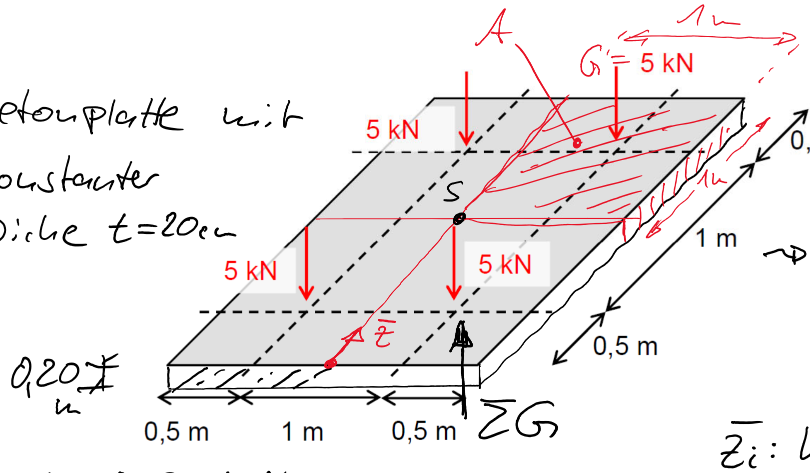
Schwerpunktbestimmung

Betonplatte mit konstanter Dicke $t = 20\,cm$

$G = 0{,}2m \cdot 25\,\frac{kN}{m^3} \cdot 1m \cdot 1m = 5\,kN$



$\bar{z}_s \cdot \Sigma G \overset{!}{=} \Sigma G_i \cdot \bar{z}_i$

$$\rightarrow \bar{z}_s = \frac{\sum_{i=1}^{n} G_i \cdot \bar{z}_i}{\sum_{i=1}^{n} G_i} \quad \} \Sigma M$$

Wichte $\gamma = 25\,\frac{kN}{m^3}$

$\bar{z}_i$: Koordinate des jeweiligen Schwerpunktes eines Teilkörpers

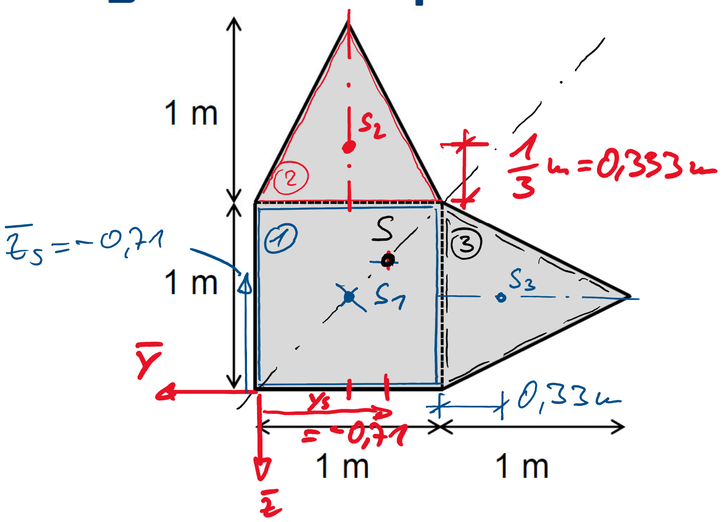
G_i: Gewicht eines Teilkörpers

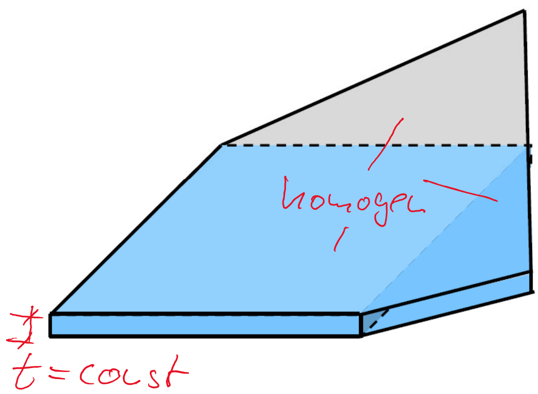
bei konstanter Dicke und Homogenität:

$G = \underbrace{\gamma}_{const} \cdot \underbrace{t}_{const} \cdot A$ — Ansichtsfläche

$$\rightarrow \bar{z}_s = \frac{\sum_{i=1}^{n} \gamma \cdot t \cdot A_i \cdot \bar{z}_i}{\sum_{i=1}^{n} \gamma \cdot t \cdot A_i} = \boxed{\frac{\Sigma A_i \cdot \bar{z}_i}{\Sigma A_i}}$$

Festigkeitslehre | Flächenschwerpunkt





$\bar{Y}_1 = -0{,}5\,m$ $\quad$ $\bar{Y}_2 = -0{,}5\,m$ $\quad$ $\bar{Y}_3 = -1{,}33\,m$

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$\bar{z}_1 = -0{,}5\,m$ $\qquad$ $\bar{z}_2 = -1{,}33\,m$ $\qquad$ $\bar{z}_3 = -0{,}5\,m$

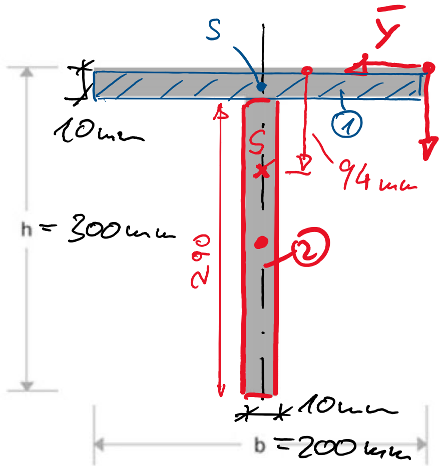
$A_1 = 1m \cdot 1m = 1m^2$ $\qquad$ $A_2 = 1m \cdot 1m/2 = 0{,}5\,m^2$ $\qquad$ $A_3 = 0{,}5\,m^2$

$$\bar{Y}_S = \frac{\sum_{i=1}^{n} A_i \cdot \bar{Y}_i}{\sum_{i=1}^{n} A_i} = \frac{A_1 \cdot \bar{Y}_1 + A_2 \cdot \bar{Y}_2 + A_3 \cdot \bar{Y}_3}{A_1 + A_2 + A_3} = \frac{\overset{m \cdot m^2}{1 \cdot (-0{,}5) + 0{,}5 \cdot (-0{,}5) + 0{,}5 \cdot (-1{,}33)}}{\underbrace{1 + 0{,}5 + 0{,}5}_{\Sigma A_i = 2{,}0\,m^2}}$$

$$= \frac{-1{,}415\,m^3}{2{,}0\,m^2} = \underline{-0{,}71\,m}$$

$$\bar{z}_S = \frac{1 \cdot (-0{,}5) + 0{,}5 \cdot (-1{,}33) + 0{,}5 \cdot (-0{,}5)}{2{,}0} = \underline{-0{,}71\,m}$$

Festigkeitslehre | Flächenschwerpunkt



$A_1 = 1cm \cdot 20cm = 20cm^2$

$\bar{y}_1 = \frac{20cm}{2} = 10cm$

$\bar{z}_1 = \frac{1cm}{2} = 0{,}5\,cm$

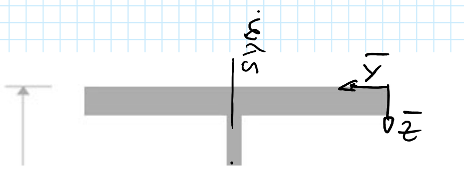
$A_2 = 1cm \cdot 29cm = 29\,cm^2$

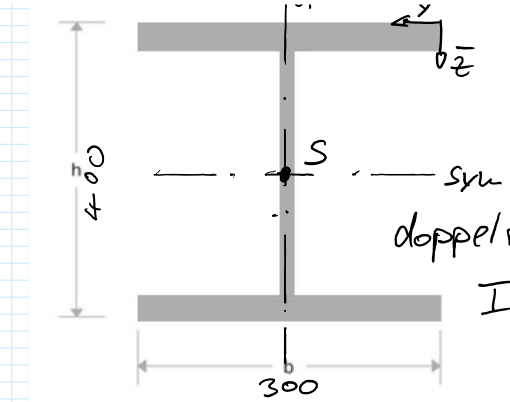
$\bar{y}_2 = 10cm$

$\bar{z}_2 = \frac{29cm}{2} + 1{,}0cm = 15{,}5cm$

$$\bar{y}_s = \frac{\sum_{i=1}^{n} A_i \cdot \bar{y}_i}{\sum_{i=1}^{n} A_i} = \frac{20 \cdot 10 + 29 \cdot 10}{20 + 29} = 10\,cm \rightarrow \text{auf d. Sym.-Achse}$$

$$\bar{z}_s = \frac{\sum_{i=1}^{n} A_i \cdot \bar{z}_i}{\sum_{i=1}^{n} A_i} = \frac{20 \cdot 0{,}5 + 29 \cdot 15{,}5}{20 + 29} = 9{,}4\ cm$$



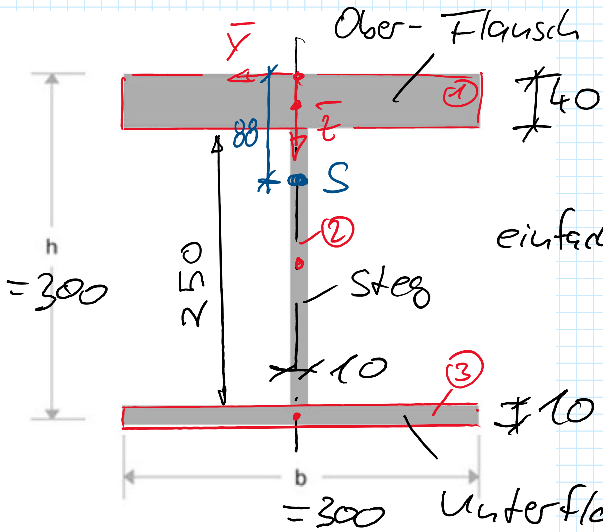


doppeltsymmetrisches I-Profil

$$\bar{y}_s = \frac{\sum_{i=1}^{n} A_i \cdot \bar{y}_i}{\sum_{i=1}^{n} A_i} = \frac{300}{2} = 150$$

$$\bar{z}_s = \frac{\sum_{i=1}^{n} A_i \cdot \bar{z}_i}{\sum_{i=1}^{n} A_i} = \frac{400}{2} = 200$$

aus Symmetrie



einfachsymmetrisch

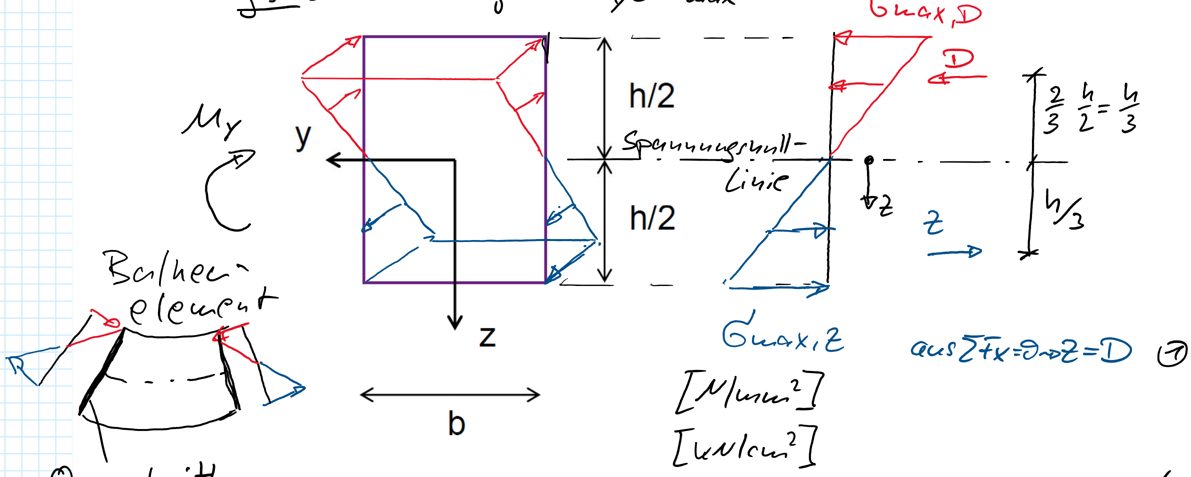
$$\bar{y}_s = \frac{\sum_{i=1}^{n} A_i \cdot \bar{y}_i}{\sum_{i=1}^{n} A_i} = 0 \quad \text{wg. Sym.}$$

$$\bar{z}_s = \frac{\sum_{i=1}^{n} A_i \cdot \bar{z}_i}{\sum_{i=1}^{n} A_i} = \frac{\underbrace{4\cdot 30}_{A_1}\cdot \underbrace{4/2}_{\bar{z}_1} + \underbrace{25}_{A_2}\cdot\underbrace{(4{,}0+\tfrac{25}{2})}_{\bar{z}_2} + \underbrace{30\cdot 1{,}0}_{A_3}\cdot(4{,}0+25+\tfrac{1{,}0}{2})}{120 + 25 + 30}$$

$$= \frac{1537{,}5\ cm^3}{175\ cm^2} \approx 8{,}8\ cm$$

Festigkeitslehre | Flächenträgheitsmoment

ges.: Zusammenhang zw. M_y & σ_{max}

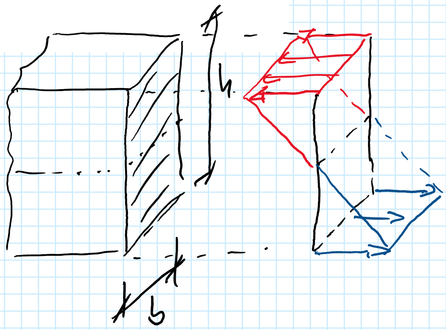


aus $\sum F_x = 0 \Rightarrow Z = D$ ①

$$M_y = D\cdot\frac{h}{3} + \underbrace{Z}_{\text{wg ①}}\cdot\frac{h}{3} = D\cdot\frac{2}{3}\cdot h \qquad ②$$

Resultierende aus Druckspannung:

$$D = \sigma_{max}\cdot\frac{h}{2}\cdot b\cdot\frac{1}{2} \qquad ③$$



③ in ②: $\underline{M_y} = \sigma_{max}\cdot\frac{hb}{4}\cdot\frac{2}{3}\cdot h = \underline{\sigma_{max}\cdot\frac{bh^2}{6}}$

$$\Rightarrow \sigma_{max} = \frac{M_y}{\frac{bh^2}{6}}$$

Widerstandsmoment $W\,[cm^3]$ eines Rechteckquerschnittes

Allgemeiner Zusammenhang zwisch $\sigma(z)$

$$\sigma(z) = \frac{M_y}{I_y}\cdot z \quad ; \quad I_y = \frac{bh^3}{12}$$ beim Rechteckquerschnitt

<u>allgemein gilt:</u>

$$\boxed{W = \frac{I_y}{z_{max}}}$$