

gesucht: Schnittkräfte N, V, M

1. Auflagerkräfte:

$$\sum \vec{F}_x \rightarrow A_x = 0$$

$$\sum \vec{M}_a = 0 = -10 \cdot 1,0 + B \cdot 5,0$$

$$\Rightarrow B = \frac{10}{5,0} = 2,0 \text{ kN}$$

$$\sum \vec{F}_z = 0 = 10 - 2,0 - A_z$$

$$\Rightarrow A_z = 8,0 \text{ kN}$$

~ 0 pos. Schnittfuß

$$\sum \vec{F}_x = 0 = N_a$$

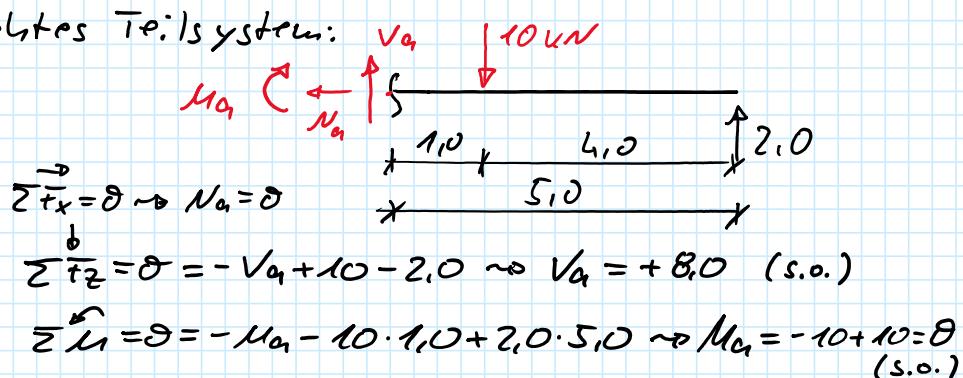
$$\sum \vec{F}_z = 0 = -V_q + 10 - 2,0 \Rightarrow V_q = +8,0 \text{ kN}$$

$$\sum \vec{M} = 0 = -M_a - 10 \cdot 1,0 + 2,0 \cdot 5,0 \Rightarrow M_a = -10 + 10 = 0$$

Schnitt rechts von A:

ragt x aus dem SU heraus
 \Rightarrow pos. Schnittfuß

Alternativ: rechtes Teilsystem:

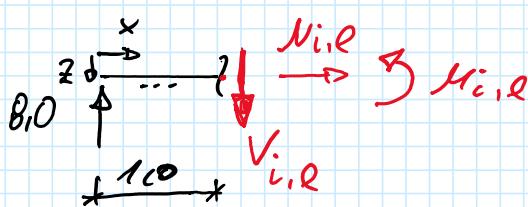


Schnitt links von i:

$$\sum \vec{F}_x \rightarrow N_{i,l} = 0$$

$$\sum \vec{F}_z = 0 = -8,0 + V_{i,l} \Rightarrow V_{i,l} = +8,0$$

$$\sum \vec{M} = 0 = -8,0 \text{ kN} \cdot 1,0 \text{ m} + M_{i,l} \Rightarrow M_{i,l} = 8,0 \text{ kNm}$$

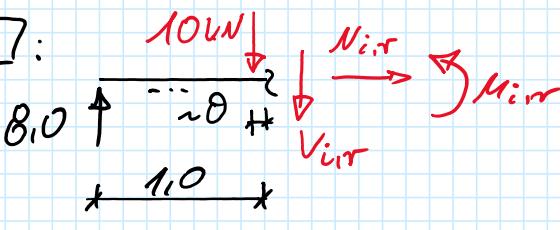


Schnitt rechts von i:

$$\sum \vec{F}_x \rightarrow N_{i,r} = 0$$

$$\sum \vec{F}_z = 0 = -8,0 + 10,0 + V_{i,r}$$

$$\Rightarrow V_{i,r} = -2,0 \text{ kN}$$



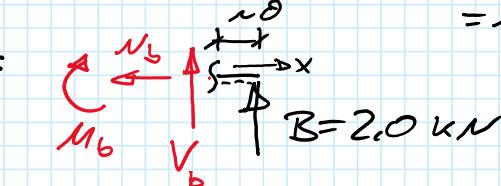
$$\rightarrow V_{i,r} = -2,0 \text{ kN}$$

$$\sum M_i = \delta = -8,0 \text{ kN} \cdot 1,0 \text{ m} + 10 \text{ kN} \cdot 0 \text{ m} + M_{i,r} \rightarrow M_{i,r} = 8,0 \text{ kNm} = M_{i,l}$$

Schnitt links von b:

rechtes Teilsystem;
x-Achse zeigt in das
Schnitthufer hinein

\rightarrow negatives SU



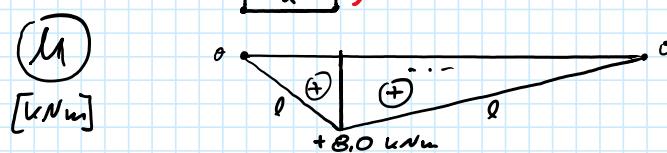
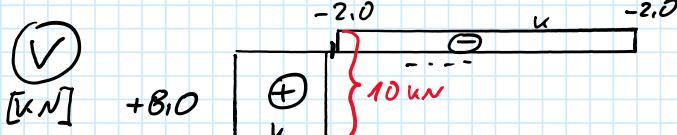
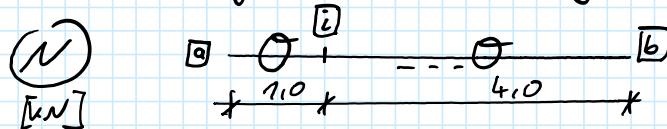
$$\sum F_x \rightarrow N_b = \delta$$

$$\sum F_z = \delta = -V_b - 2,0 \text{ kN}$$

$$\rightarrow V_b = -2,0 \text{ kN}$$

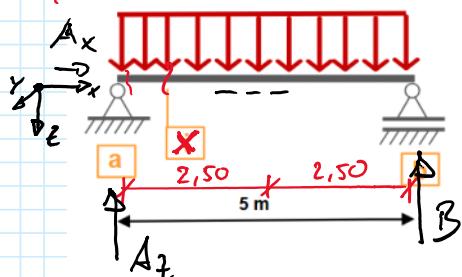
$$\sum M_i = \delta = -M_b + B \cdot \delta \rightarrow M_b = \delta$$

Darstellung der Schnittgrößen:



$$R = 50 \text{ kN}$$

$$q(x) = 10 \text{ kN/m} = \text{const}$$



Einfeldträger mit Gleichstreckenlast

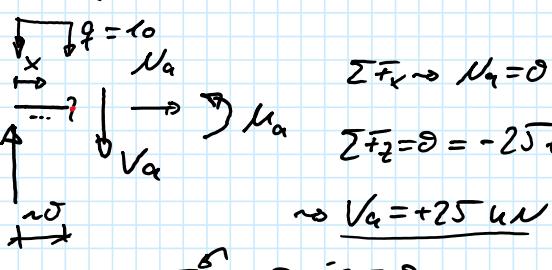
$$\text{Resultierende bilde: } R = 10 \frac{\text{kN}}{\text{m}} \cdot 5 \text{ m} = 50 \text{ kN}$$

Auflager:

$$\sum M_{a,x} = \delta = -50 \cdot 2,50 + B \cdot 5,0 \rightarrow B = 25 \text{ kN}$$

$$\sum F_z = \delta = -A_z + 50 - 25 \rightarrow A_z = 25$$

$$\sum F_x = \delta \rightarrow A_x = 0$$



$$\sum F_x \rightarrow N_a = 0$$

$$\sum F_z = \delta = -25 + 10 \cdot 0 + V_a$$

$$\rightarrow V_a = +25 \text{ kN}$$

Schnitt rechts von a:

$$A_z = 25$$

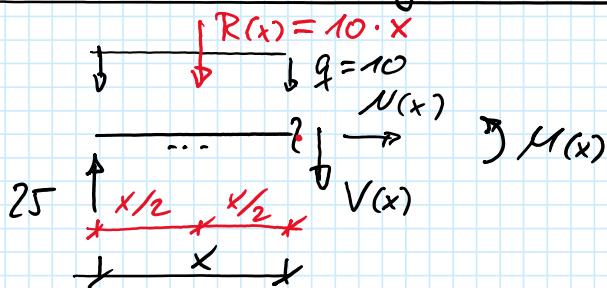
$$A_x = 0$$

$$\sum M = 0 \quad \text{at } x = 0$$

$$\approx V_a = +25 \text{ kN}$$

$$\sum M = 0 = 25 \cdot 0 + M_a \Rightarrow M_a = 0$$

Schmitt bei beliebiger Koordinate x:



$$\sum F_x \approx N(x) = 0$$

$$\sum F_z = 0 = -25 + R(x) + V(x)$$

$$\Rightarrow V(x) = 25 - 10 \cdot x \quad \text{Geraden-gleichung}$$

$$\sum M = 0 = -25 \cdot x + R(x) \cdot \frac{x}{2} + M(x)$$

$$\Rightarrow M(x) = 25 \cdot x - 10 \cdot x \cdot \frac{x}{2} = 25x - 5x^2$$

quadratische Parabel

Auswertung:

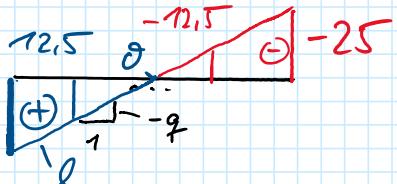
x	$\frac{x}{L}$	$V(x) [\text{kN}]$	$M(x) [\text{kNm}]$
0	0	25	0
1,25	$\frac{1}{4}$	12,5	23,44
2,5	$\frac{1}{2}$	0	31,25
3,75	$\frac{3}{4}$	-12,5	23,44
5,0	1,0	-25	0

Verläufe:

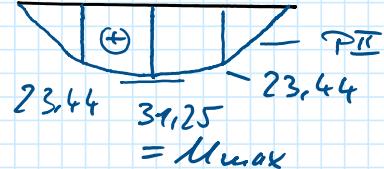
$$(N) [\text{kN}]$$

$$0 \quad 0$$

$$(V) [\text{kN}]$$



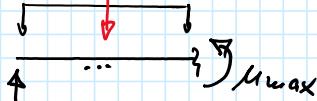
$$(M) [\text{kNm}]$$



Hier:

$$V_{\max} = \frac{q \cdot L}{2}$$

$$R = q \cdot L / 2$$



M_{\max} in Feldmitte:

$$A = \frac{q \cdot L}{2}$$

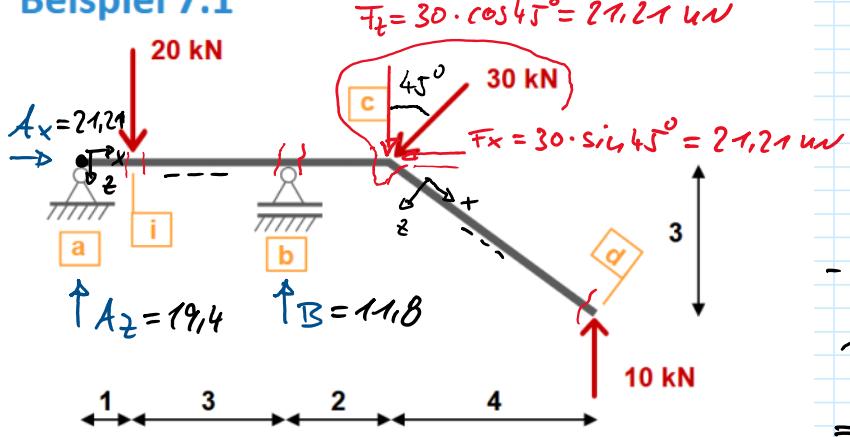
$$+ \frac{L/4}{L/2} \quad \frac{L/4}{L/2}$$

$$\sum M = 0 = -A \cdot \frac{L}{2} + R \cdot \frac{L}{4} + M_{\max}$$

$$\Rightarrow M_{\max} = \frac{q \cdot L}{2} \cdot \frac{L}{2} - \frac{q \cdot L}{2} \cdot \frac{L}{4}$$

$$M_{\max} = \frac{q L^2}{8}$$

Beispiel 7.1



ges.: N, V, M

$$\sum F_x = \delta = A_x - 21,21$$

$$\Rightarrow A_x = 21,21 \text{ kN}$$

$$\sum M_a = \delta = -20 \cdot 1,0 + 3 \cdot 4,0$$

$$- 21,21 \cdot 6,0 + 21,21 \cdot 8 + 10 \cdot 10,0$$

$$\Rightarrow B = 11,8 \text{ kN}$$

$$\sum F_z = \delta$$

$$= - A_z + 20 - 11,8 + 21,21 - 10$$

$$\Rightarrow A_z = 19,4 \text{ kN}$$

$$\boxed{a}: \begin{array}{c} A_x = 21,21 \\ N_a \\ V_a = 19,4 \\ 19,4 \\ \hline \end{array} \quad \sum F_x = \delta = 21,21 + N_a \Rightarrow N_a = -21,21 \text{ kN}$$

links von \boxed{i} :

$$\begin{array}{c} 21,21 \\ \hline \begin{array}{c} 19,4 \\ \hline 1,0 \end{array} \end{array} \quad \begin{array}{c} N_{i,l} \\ V_{i,l} \\ \hline \end{array} \quad \begin{array}{c} M_{i,l} \\ M_{i,r} \\ \hline \end{array} \quad \begin{array}{c} \sum F_x = N_{i,l} = -21,21 \text{ kN} \\ \sum F_z = V_{i,l} = +19,4 \text{ kN} \\ \sum M = \delta = 21,21 \cdot 8 - 19,4 \cdot 1,0 + M_{i,l} \\ \Rightarrow M_{i,l} = 19,4 \text{ kNm} \end{array}$$

rechts von \boxed{i} :

$$\begin{array}{c} 20 \\ \hline \begin{array}{c} 19,4 \\ \hline 1,0 \end{array} \end{array} \quad \begin{array}{c} N_{i,r} \\ V_{i,r} \\ \hline \end{array} \quad \begin{array}{c} M_{i,r} \\ M_{i,r} \\ \hline \end{array} \quad \begin{array}{c} N_{i,r} = N_{i,l} = -21,21 \text{ kN} \\ \sum F_z = -19,4 + 20 + V_{i,r} \\ \Rightarrow V_{i,r} = -0,6 \text{ kN} \\ M_{i,r} = M_{i,l} = 19,4 \text{ kNm} \end{array}$$

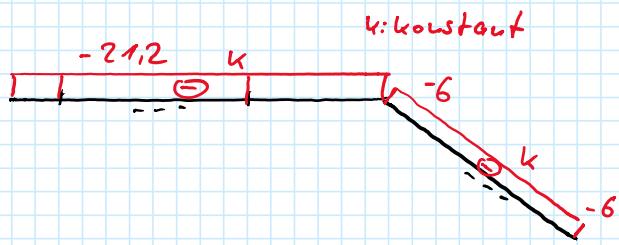
links von \boxed{b} :

$$\begin{array}{c} 21,21 \\ \hline \begin{array}{c} 19,4 \\ \hline 1,0 \end{array} \end{array} \quad \begin{array}{c} 20 \\ \hline \begin{array}{c} 3,0 \\ \hline \end{array} \end{array} \quad \begin{array}{c} N_{b,l} \\ V_{b,l} \\ \hline \end{array} \quad \begin{array}{c} M_{b,l} \\ M_{b,r} \\ \hline \end{array} \quad \begin{array}{c} N_{b,r} = N_{i,r} = -21,21 \\ V_{b,r} = V_{i,r} = -0,6 \\ \sum M = \delta = -19,4 \cdot 4,0 + 20 \cdot 3,0 + M_{b,r} \\ \Rightarrow M_{b,r} = 17,6 \text{ kNm} \end{array}$$

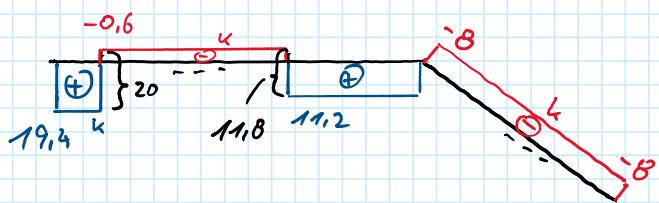
rechts von \boxed{b} :

$$\begin{array}{c} 21,21 \\ \hline \begin{array}{c} 19,4 \\ \hline \end{array} \end{array} \quad \begin{array}{c} 20 \\ \hline \begin{array}{c} 11,8 \\ \hline \end{array} \end{array} \quad \begin{array}{c} N_{b,r} = -21,21 \\ V_{b,r} \\ \hline \end{array} \quad \begin{array}{c} M_{b,r} = M_{b,l} \\ = 17,6 \\ \hline \end{array} \quad \begin{array}{c} \sum F_z = \delta = -19,4 + 20 - 11,8 + V_{b,r} \end{array}$$

N
[kN]



\checkmark
[kN]



M
[kNm]

