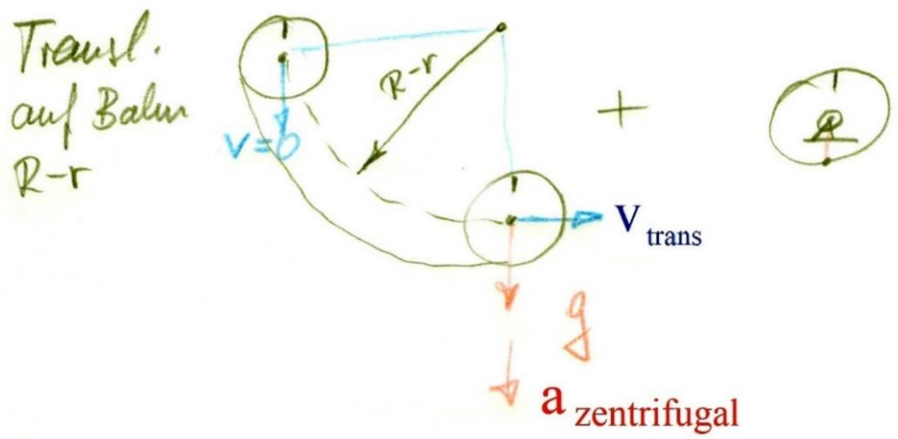
Musterlösungen: Aufgaben Prof. Zurmühl/Wörnle/Maquerre

Energiesatz + Beschleunigungen + Leistung

1.

Musterlösung Aufgabe Energiesatz + Beschleunigungen

Translation Rotation



Energiesatz

$$\cancel{m}\cdot g\cdot(R-r) = \frac{1}{2} m v^2 + \frac{1}{2} J \omega^2 \quad ; \quad v=\omega\cdot r$$

$$= \frac{1}{2} m v^2 + \frac{1}{2}\cdot\frac{1}{2} m \cancel{r^2} \cdot \frac{v^2}{\cancel{r^2}}$$

$$= \frac{1}{2} m v^2 + \frac{1}{4} m v^2$$

$$= \frac{3}{4} \cancel{m} v^2$$

$$v_{tr.} = \sqrt{\frac{4}{3} g (R-r)}$$

Beschleunigungen

$$a_{n,zentr} = \frac{v_{trans}^2}{R-r} = \frac{\frac{4}{3} g (R-r)}{R-r} = \frac{4}{3} g$$

$$a_{ges} = g + a_{n,zentr} = \frac{3}{3} g + \frac{4}{3} g = \frac{7}{3} g$$

$$F = \frac{7}{3} m \cdot g$$

2.

Arbeit / Leistung   Dirmahl   S. 12 Nr. 5

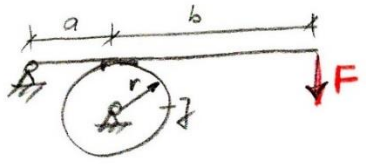
$a = 0{,}6\,m \qquad r = 0{,}3\,m$

$b = 1{,}2\,m$

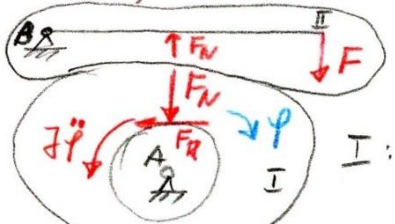
$J = 3\,kg\,m^2$

$F = 100\,N$

$n_1 = 2600\,\frac{1}{min}\ ;\ n_2 = 2000\,\frac{1}{min}$

a) 

Ges: μ



$\mathrm{II}: \curvearrowleft B: F_N \cdot a - F(a+b) = 0$

$F_N = F \cdot \frac{a+b}{a} = 100\,N\,\frac{1{,}8}{0{,}6} = 300\,N$

$\mathrm{I}: \curvearrowleft A: J\ddot{\varphi} + F_R \cdot r = 0$

$\ddot{\varphi} = -\frac{F_R \cdot r}{J} = -\frac{\mu \cdot F_N \cdot r}{J}$

$\ddot{\varphi} = -\mu \cdot \frac{300\,N \cdot 0{,}3\,m}{3\,kg\,m^2} = -\mu \frac{300\,\cancel{kg\,m} \cdot 0{,}3\,\cancel{m}}{3\,\cancel{kg\,m^2}\,s^2}$

$\ddot{\varphi} = -\mu \cdot 30\,\frac{1}{s^2}$

Coulomb: $F_R = \mu \cdot F_N$;

$\omega = \dot{\varphi} = -\mu \cdot 400\,\frac{1}{s^2} \cdot t$

$\omega_1 = 2\pi \cdot n_1 \mathrel{\hat{=}} (t = 0)$

$\omega_2 = \omega_1 - \mu \cdot 30\,\frac{1}{s^2} \cdot t$

$\omega_2 = 2\pi n_1 - \mu \cdot 30\,\frac{1}{s^2} \cdot 40\,s$

$2\pi \cdot n_2 = 2\pi \cdot n_1 - \mu \cdot 1200\,\frac{1}{s}$

$\underbrace{2\pi(n_2 - n_1)}_{-2\pi \cdot 10\,\frac{1}{s}} = -\mu \cdot 1200\,\frac{1}{s} \quad \Rightarrow \mu = \frac{20\pi}{1200} = \frac{\pi}{60} = 0{,}0524$

b) $F = 150\,N \quad ; \ n = 2330\,\frac{1}{min}$

$F_N = 150 \cdot 3 = 450\,N$

$M = F_R \cdot r = \mu \cdot F_N \cdot r = 0{,}0524 \cdot 450\,N \cdot 0{,}3\,m = 7{,}074\,Nm$

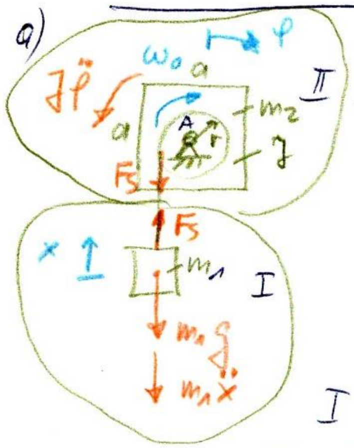
$P = M \cdot \omega = M \cdot 2\pi \cdot n = 7{,}074\,Nm \cdot 2\pi \cdot \frac{2330}{60}\,\frac{1}{s}$

$P = 1{,}726\,kW$

3.

Zurmühl Kinetik - Massenträgheit

a)



Würfel (Kantenlänge a)

Bei $t=0$ wirkt ω_0

$$J_{Quader} = \frac{m_2}{12}\left(a^2+a^2\right)$$

$$J = \frac{m_2 a^2}{6}$$

Zw. bed: $\ddot{x} = r\cdot\ddot{\varphi}$

$$I: \quad F_S = m_1\cdot g + m_1\ddot{x}$$

$$II: \quad (A): \; J\cdot\ddot{\varphi} + F_S\cdot r = 0$$

$$J\cdot\ddot{\varphi} + \left(m_1 g + m_1\ddot{x}\right)\cdot r = 0$$

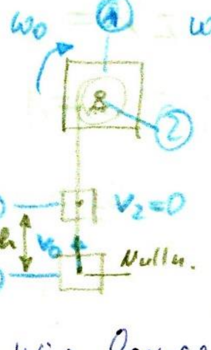
$$J\cdot\frac{\ddot{x}}{r} + m_1\ddot{x}\cdot r = -m_1\cdot g\cdot r$$

$$\ddot{x}\left(\frac{J}{r} + m_1 r\right) = -m_1 g r$$

$$\ddot{x} = -g\left(\frac{m_1}{\frac{J}{r^2} + m_1}\right)$$

b) Wie hoch steigt m_1 bis zum Stillstand

Energiesatz



$$\frac{1}{2} m_1 v_0^2 + \frac{1}{2} J\omega_0^2 = m_1 g\cdot h \quad ; \quad v_0 = \omega_0\cdot r$$

$$\frac{1}{2} m_1 \omega_0^2\cdot r^2 + \frac{1}{2} J\omega_0^2 = m_1 g h$$

$$h = \frac{\frac{1}{2}\omega_0^2 r^2 + \frac{\frac{1}{2} J\omega_0^2}{m_1}}{g} = \frac{\omega_0^2}{2g}\left(r^2 + \frac{J}{m_1}\right)$$

c) Wie lange dauert das Steigen?

$$\dot{x} = -g\frac{m_1}{\left(\frac{J}{r^2} + m_1\right)}\cdot t + C_1 \quad ; \quad \dot{x}_{(t=0)} = v_0 = \omega_0\cdot r = C_1$$

$$\dot{x} = \frac{-g m_1}{\left(\frac{J}{r} + m_1\right)} t + \omega_0\cdot r$$

Für $\dot{x} = 0 \Rightarrow t = \frac{\omega_0\cdot r\left(\frac{J}{r} + m_1\right)}{g\cdot m_1}$

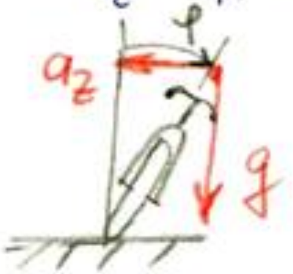
4 Radfahrer

Zurückl S. 13 Nr. 1

$R = 10\,m$ | **Ges**: Neigungswinkel φ

$v = 5\,\frac{m}{s}$ | Haftreibungskoef. μ

$$a_z = \frac{v^2}{R} = \frac{25\,\frac{m^2}{s^2}}{10} = 2{,}5\,\frac{m}{s^2}$$



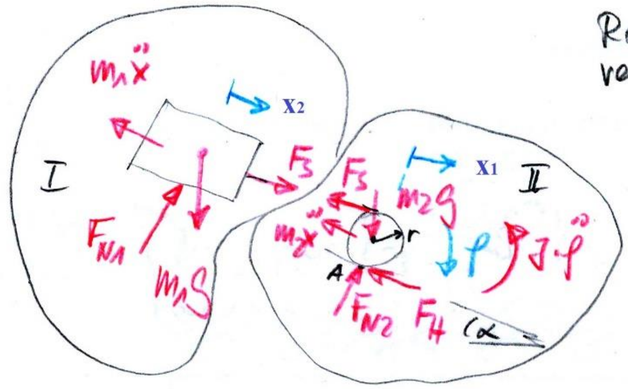
$$\tan\varphi = \frac{a_z}{g} = \frac{v^2}{R \cdot g} = \frac{2{,}5\,\frac{m}{s^2}}{9{,}81\,\frac{m}{s^2}} = 0{,}255$$

$$\varphi = 14{,}3^\circ$$

$$\mu_0 = \tan\varphi = 0{,}255$$

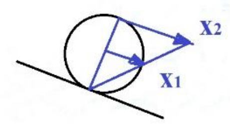
5.

Woernle 13.3



Rollreibung wird vernachlässigt

Ges: F_S, $\ddot{x}$



$$\text{I}: \curvearrowleft A): F_S\cdot 2r + m_2\ddot{x}_1\cdot r + J\ddot{\varphi} - m_2 g\sin\alpha\cdot r = 0 \qquad (1)$$

$$\text{II}: \searrow: F_S - m_1\ddot{x}_2 + m_1 g\sin\alpha = 0$$

$$\Rightarrow F_S = m_1\ddot{x}_2 - m_1 g\sin\alpha \qquad (2)$$

$$(2) \text{ in } (1): 2(m_1\ddot{x}_2 - m_1 g\sin\alpha) + m_2\ddot{x}_1 + J\frac{\ddot{\varphi}}{r} - m_2 g\sin\alpha = 0$$

Zwangsbed.: $x = r\cdot\varphi \Rightarrow \boxed{\ddot{\varphi} = \frac{\ddot{x}_1}{r}}$; $\boxed{\ddot{x}_2 = 2\ddot{x}_1}$

Massenträgh.m.: $J = \frac{1}{2}m_2r^2$

$$2m_1\ddot{x}_2 - 2m_1 g\sin\alpha + m_2\ddot{x}_1 + \frac{1}{2}m_2\cancel{r^2}\cdot\frac{\ddot{x}_1}{\cancel{r\cdot r}} - m_2 g\sin\alpha = 0$$

$$\ddot{x}_1\left(4m_1 + m_2 + \frac{m_2}{2}\right) = (2m_1 + m_2)\, g\sin\alpha$$

$$\ddot{x}_1(4m_1 + 1{,}5m_2) = g\sin\alpha\,(2m_1 + m_2)$$

$$\ddot{x}_1 = g\sin\alpha\,\frac{2m_1 + m_2}{4m_1 + 1{,}5m_2}$$

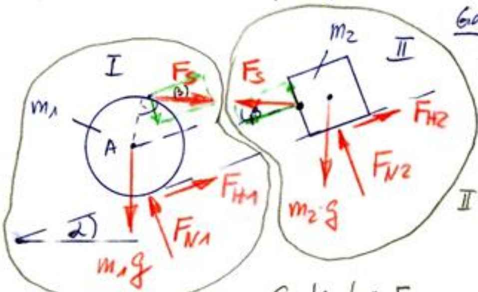
$$F_S = 2m_1\cdot g\sin\alpha\,\frac{2m_1 + m_2}{4m_1 + 1{,}5m_2} - m_1 g\sin\alpha = m_1 g\sin\alpha\left[\frac{4m_1 + 2m_2}{4m_1 + 1{,}5m_2} - \frac{4m_1 + 1{,}5m_2}{4m_1 + 1{,}5m_2}\right]$$

$$= m_1 g\sin\alpha\left[\frac{0{,}5m_2}{4m_1 + 1{,}5m_2}\right] = g\sin\alpha\left[\frac{0{,}5\,m_1m_2}{4m_1 + 1{,}5m_2}\right]$$

$$F_S = g\sin\alpha\left[\frac{m_1m_2}{8m_1 + 3m_2}\right] = g\sin\alpha\left[\frac{1}{\frac{8}{m_2} + \frac{3}{m_1}}\right]$$

6-

Wörnle : Haftung Nr. 3.4



Ges: α_{max} für Haftung

$\mathrm{I}: \curvearrowleft A: -F_S \cdot r + F_{H1} \cdot r = 0$

$F_S = F_{H1}$ (1)

$\nearrow: F_S \cdot \cos\beta + F_{H1} - m_1 g \cdot \sin\alpha = 0$ (7)

$\nwarrow: F_{N1} - m_1 g \cdot \cos\alpha - F_S \cdot \sin\beta = 0$ (2)

$\mathrm{II}: \nearrow: F_{H2} - m_2 g \cdot \sin\alpha - F_S \cdot \cos\beta = 0$ (3)

$\nwarrow: F_{N2} - m_2 g \cdot \cos\alpha + F_S \cdot \sin\beta = 0$ (4)

Coulomb: $F_{H1} = \mu_0 \cdot F_{N1}$; $F_{H2} = \mu_0 \cdot F_{N2}$ (5);(6)

Walze

(2): $F_S \cdot \sin\beta = F_{N1} - m_1 g \cos\alpha = F_{H1} \cdot \sin\beta = \mu_0 \cdot F_{N1} \cdot \sin\beta$

$$F_{H1} = \frac{F_{N1} - m_1 g \cos\alpha}{\sin\beta} = \mu_0 \cdot F_{N1}$$

$F_S \cdot \cos\beta + F_S = m_1 g \sin\alpha$

$F_S (1 + \cos\beta) = m_1 g \sin\alpha$

$F_S = F_{H1} = m_1 g \frac{\sin\alpha}{1 + \cos\beta}$

$F_{H1} = \mu_0 F_{N1} = \mu_0 (m_1 g \cos\alpha + F_S \cdot \sin\beta)$

$F_{H1} = \mu_0 \cdot m_1 g \cos\alpha + \mu_0 \cdot F_{H1} \cdot \sin\beta$

$F_{H1} (1 - \mu_0 \sin\beta) = \mu_0 m_1 g \cos\alpha$

$$\frac{\mu_0 m_1 g \cos\alpha}{1 - \mu_0 \sin\beta} = m_1 g \frac{\sin\alpha}{1 + \cos\beta} \Rightarrow \frac{\sin\alpha}{\cos\alpha} = \frac{1 + \cos\beta}{1 - \mu_0 \sin\beta}$$

$$\tan\alpha = \frac{1 + \cos\beta}{1 - \mu_0 \sin\beta} = \frac{0{,}134}{1 - \mu_0\, 0{,}5}$$

Klotz

$$F_S = F_{H1} = m_1 g \frac{\sin\alpha}{1 + \cos\beta}$$

$F_{H2} = \mu_0 \cdot F_{N2}$

$m_2 g \sin\alpha + F_S \cdot \cos\beta = \mu_0 (m_2 g \cos\alpha - F_S \cdot \sin\beta)$

$$m_2 g \cdot \sin\alpha + m_1 g \frac{\sin\alpha \cdot \cos\beta}{1 + \cos\beta} = \mu_0 \cdot \left(m_2 g \cos\alpha - m_1 g \frac{\sin\alpha \cdot \sin\beta}{1 + \cos\beta}\right) \quad | : \sin\alpha$$

$$m_2 g + m_1 g \frac{\cos\beta}{1 + \cos\beta} = \mu_0 \cdot m_2 g \cdot \frac{1}{\tan\alpha} - \mu_0 m_1 g \frac{\sin\beta}{1 + \cos\beta} \quad | : m_2 g$$

$$1 + \frac{m_1 g}{m_2 g} \frac{\cos\beta}{1 + \cos\beta} + \frac{m_1 g}{m_2 g} \frac{\sin\beta \cdot \mu_0}{1 + \cos\beta} = \mu_0 \frac{1}{\tan\alpha}$$

$$\tan\alpha = \frac{\mu_0}{1 + \frac{m_1 g}{m_2 g} \left(\frac{\cos\beta + \mu_0 \sin\beta}{1 + \cos\beta}\right)} = \frac{\mu_0}{1 + \frac{m_1 g}{m_2 g} \left(\frac{\mu_0 + \frac{\cos 30^\circ}{\sin 30^\circ}}{\frac{1}{\sin 30^\circ} + \frac{\cos 30^\circ}{\sin 30^\circ}}\right)}$$

$$\tan\alpha = \frac{\mu_0}{1 + \frac{m_1 g}{m_2 g} \left(\frac{\mu_0 + 1{,}732}{2 + 1{,}732}\right)}$$

$\left(\frac{m_1}{m_2} = 1\right)$

a) Walze ; $\mu_0 = 0{,}1$

$$\tan\alpha = \frac{0{,}134}{1 - 0{,}1 \cdot 0{,}5} = 0{,}14$$

$\underline{\alpha = 8{,}03^\circ}$

b) Klotz

$$\tan\alpha = \frac{0{,}1}{1 + \frac{m_1 g}{m_2 g} \left(\frac{0{,}1 + 1{,}732}{2 + 1{,}732}\right)}$$

$$\tan\alpha = \frac{0{,}1}{1 + \frac{m_1 g}{m_2 g} \cdot 0{,}49}$$

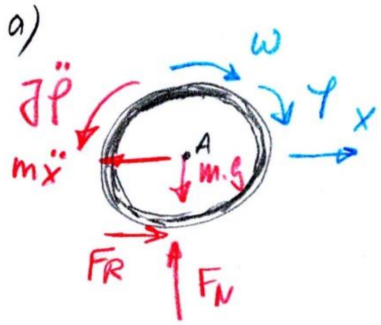
$\tan\alpha = 0{,}067$

$\underline{\alpha = 3{,}84^\circ}$ (Grenzwinkel)

7.

M/W S.21 Nr. 13.12

a)



$$J_{Hz} = \frac{m\,(r_a^2 + r_i^2)}{2} \; ; \; r_i \approx r_a = r$$
$$J_{Hz} = \frac{m\,2r^2}{2} = mr^2$$

$$\curvearrowleft A: \; J\ddot{\varphi} + F_R \cdot r = 0$$
$$\ddot{\varphi} = -\frac{F_R \cdot r}{J} = -\frac{\mu \cdot m \cdot g \cdot r}{J}$$
$$\ddot{\varphi} = -\frac{\mu \cancel{m} g \cancel{r}}{\cancel{m} r^{\cancel{2}}} = -\frac{\mu g}{r}$$
$$\dot{\varphi} = \omega = \int \ddot{\varphi}\, dt$$
$$\dot{\varphi} = -\int \frac{g}{r}\, dt = -\frac{g}{r} \cdot t + C_1$$
$$\dot{\varphi}_{(t=0)} = \omega = C_1$$
$$\dot{\varphi} = -\frac{\mu g}{r} \cdot t + \omega$$

$$\rightarrow: \; -m\ddot{x} + F_R = 0$$
$$m\ddot{x} = m \cdot g \cdot \mu$$
$$\ddot{x} = g\mu$$
$$\dot{x} = \int \ddot{x}\, dt = g\mu \cdot t + C_2$$
$$v_{(t=0)} = 0 \Rightarrow C_2 = 0$$
$$\dot{x} = g\mu \cdot t$$

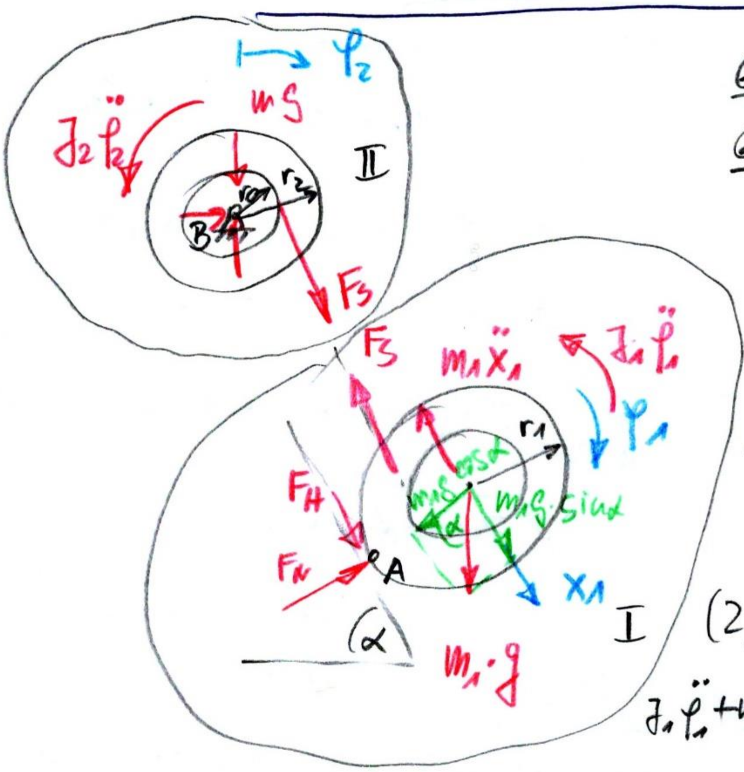
Für reines Rollen gilt:

$$v = \omega \cdot r$$
$$g\mu \cdot t = -\left(\frac{\mu g \cdot t}{r} + \omega\right) \cdot r$$
$$2g\mu \cdot t = \omega \cdot r$$
$$t = \frac{\omega\, r}{2\mu g}$$

b)

$$\dot{x} = g \cdot \mu \cdot t = \frac{\cancel{g} \cdot \cancel{\mu} \cdot \omega \cdot r}{2\cancel{\mu g}} = \frac{\omega \cdot r}{2}$$
$$\dot{\varphi} = -\frac{\cancel{\mu g}}{\cancel{r}} \cdot \frac{\omega \cdot \cancel{r}}{2\cancel{\mu g}} + \omega = \frac{\omega}{2}$$

M/W. S.21. Nr. 13.9



Geg: $m_1;\ J_1,\ J_2,\ \alpha;\ r_0,\ r_1,\ r_2$

Ges: $\ddot{x}_1,\ \ddot{\varphi}_1$

I: $\curvearrowleft A$:

$$J_1\ddot{\varphi}_1 + m_1\ddot{x}_1\cdot r_1 - m_1 g\cdot\sin\alpha\cdot r_1 + F_S(r_1 - r_0) = 0$$

II: $\curvearrowleft B$: $J_2\ddot{\varphi}_2 - F_S\cdot r_0 = 0$

$$F_S = \frac{J_2\ddot{\varphi}_2}{r_0} \qquad (2)$$

(2) in (1):

$$J_1\ddot{\varphi}_1 + m_1\ddot{x}_1\cdot r_1 + \frac{J_2\cdot\ddot{\varphi}_2}{r_0}(r_1 - r_0) = m_1 g\sin\alpha\cdot r_1$$

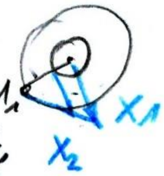
Zwangsbed.: $\ddot{x}_1 = r_1\cdot\ddot{\varphi}_1$; $\ddot{x}_2 = \ddot{\varphi}_2\cdot r_0$;

$$\ddot{\varphi}_2 = \frac{\ddot{x}_2}{r_0} = \ddot{\varphi}_1\cdot r_1\left(\frac{r_1 - r_0}{r_1\cdot r_0}\right)$$

$$\frac{\ddot{x}_1}{r_1} = \frac{\ddot{x}_2}{r_1 - r_0}$$

$$\ddot{x}_2 = \frac{r_1 - r_0}{r_1}\cdot\ddot{x}_1$$

$$\ddot{x}_2 = \ddot{x}_1\left(1 - \frac{r_0}{r_1}\right)$$



$$J_1\cdot\ddot{\varphi}_1 + m_1\cdot r_1\cdot\ddot{\varphi}_1\cdot r_1 + J_2\cdot\ddot{\varphi}_1\cdot r_1\left(\frac{r_1 - r_0}{r_1\cdot r_0}\right)\frac{(r_1 - r_0)}{r_0} = m_1 g\, r_1\sin\alpha$$

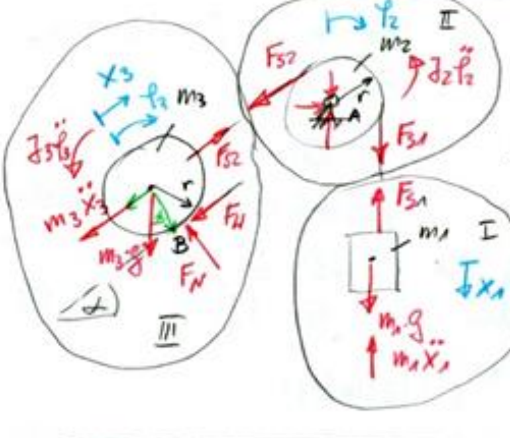
$$\ddot{\varphi}_1\cdot\left[J_1 + m_1 r_1^2 + J_2\cdot r_1\left(\frac{(r_1 - r_0)^2}{r_1\cdot r_0^2}\right)\right] = m_1\cdot g\, r_1\sin\alpha$$

$$\ddot{\varphi}_1 = \frac{m_1 g\, r_1\sin\alpha}{J_1 + J_2\cdot\frac{(r_1 - r_0)^2}{r_0^2} + m_1 r_1^2}$$

$$\ddot{x} = r_1\cdot\ddot{\varphi}_1 = \frac{m_1 g\, r_1^2\sin\alpha}{J_1 + J_2\cdot\frac{(r_1 - r_0)^2}{r_0^2} + m_1 r_1^2}$$

9.

MIW. S.21. Nr. 13.10



$m_2 = m_3 = m$

$I: \uparrow: F_{S1} = m_1 g - m_1 \ddot{x}_1$ **(01)**

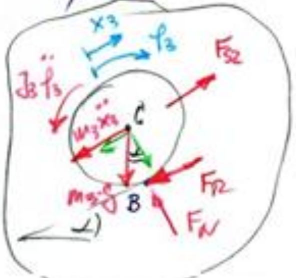
$II: \circlearrowleft A: F_{S2} \cdot r - F_{S1} \cdot r + J_2 \ddot{\varphi}_2 = 0$ **(02)**

$III: \circlearrowleft B: m_3 \ddot{x}_3 \cdot m_3\, g \sin\alpha\, r + J_3 \ddot{\varphi}_3 - F_{S2} \cdot r = 0$ (3)

Zw. bed.: $\ddot{x}_1 = \ddot{x}_3 = \ddot{x}$

$\ddot{\varphi}_2 = \frac{\ddot{x}}{r}$

b) Rolle 3 rutscht durch mit $F_R = \mu \cdot F_N$ (Bewegung nach oben)



$\ddot{x}_1 = \ddot{x}_3$; φ_3 keine Rollbed.

$\circlearrowleft C: J \ddot{\varphi}_3 - \mu \cdot m \cdot g \cdot \cos\alpha \cdot r = 0$

$\frac{1}{2} \cancel{m} \cdot r^{\cancel{2}} \cdot \ddot{\varphi}_3 = \mu \cdot \cancel{m} \cdot g \cdot \cos\alpha \cdot \cancel{r}$

$$\ddot{\varphi}_3 = \frac{2 \mu g \cos\alpha}{r} \qquad (5)$$

$\circlearrowleft B: m \cdot \ddot{x} \cdot r + m_3 g \sin\alpha \cdot r + J_3 \ddot{\varphi}_3 - F_{S2} \cdot r = 0$ (1)

$\nearrow: F_{S2} - m \cdot \ddot{x} - m_3 g \sin\alpha + F_R = 0$ (2)

Coulomb: $F_R = \mu \cdot F_N$

$\nwarrow: F_N = m \cdot g \cdot \cos\alpha$

(1) und (2): $m_1 g \sin\alpha + \frac{J_3 \cdot \ddot{\varphi}_3}{r} - F_{S2} = F_{S2} - m_3 g \sin\alpha - \mu \cdot m \cdot g \cdot \cos\alpha$

$2 F_{S2} = m_1 g \sin\alpha + \frac{J_3 \cdot \ddot{\varphi}_3}{r} + m_3 g \sin\alpha + m_3 g \cdot \cos\alpha \cdot \mu$ **(4)**

$(4)+(5)$ in (3): $m_3 \ddot{x}_3 \cdot r + J_3 \cdot \ddot{\varphi}_3 - (m_1 g - m_1 \ddot{x}) r + \frac{J_2 \cdot \ddot{\varphi}_2}{r} = -m g \sin\alpha \cdot r$

$m \ddot{x} + \frac{1}{2} m r^2 \frac{2 \mu g \cos\alpha}{r^2} - m_1 g + m_1 \ddot{x} + \frac{1}{2} m r^2 \frac{\ddot{x}}{r^2} = -m g \sin\alpha$

$\ddot{x} \left[m + m_1 + 0{,}5 m \right] = m_1 g - m g \sin\alpha - m \mu g \cos\alpha$

$\ddot{x} = \frac{m_1 g - m \sin\alpha) - m \mu g \cos\alpha}{\frac{3}{2} m + m_1} \qquad | : m_1$

$$\ddot{x} = \frac{g\left[1 - \frac{m}{m_1} \sin\alpha - \frac{m}{m_1} \mu g \cos\alpha\right]}{1 + \frac{3}{2} \frac{m}{m_1}} = g\, \frac{1 - \frac{m}{m_1}(\sin\alpha - \mu \cos\alpha)}{1 + \frac{3}{2} \frac{m}{m_1}}$$

b) $\ddot{x} = 0{,}7\, g$; $\ddot{\varphi} = 0{,}5\, g/r$