Energy Economics

Fachbereich 2 Informatik und Ingenieurwissenschaften

Reaction of demand on price changes

- We assume a normal good leading to a decreasing demand with increasing prices
- How strong is the reaction of demand on price changes?
- For comparability we focus on relative changes

In order to measure the “sensitivity” of demand with respect to a price change, we use the (price) elasticity of demand to answer the question:

> What is the change of demand on a percentage basis,
> if the price changes 1 %?

Price elasticity of demand

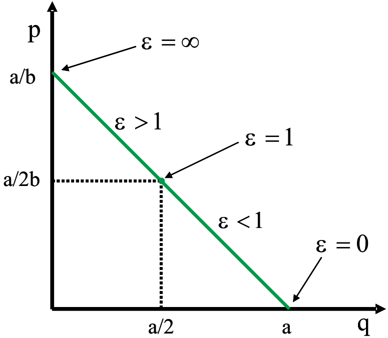
$$\varepsilon_p = \left| \frac{\%\text{ change of demand}}{\%\text{ change of prices}} \right| = -\frac{\Delta x / x}{\Delta p / p} = \frac{\Delta x}{\Delta p} \frac{p}{x}$$

or for a continuously differentiable demand function

$$\varepsilon_p = -\frac{d\,D(p)}{dp} \cdot \frac{p}{D(p)}$$

- Since the demand for normal goods decreases with increasing prices, the slope (respectively the derivative $d\,D(p)/dp$) is always negative. By definition this compensated by a minus.

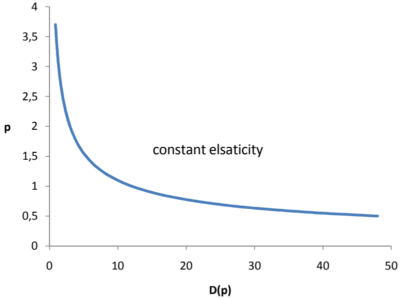
Elasticity of demand along the price curve



The elasticity of demand changes along the demand curve although the slope is constant. It varies between $\varepsilon = 0$ (for $p = 0$) and $\varepsilon = \infty$ (for $D(p) = 0$).

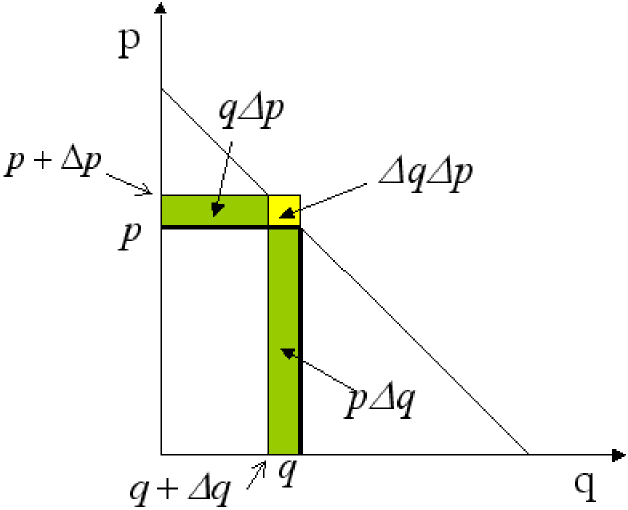
Constant elasticity of demand

Price elasticity along the following curve is constant.



- A high price with low demand is compensated by a small value for the derivative $D'(p)$.
- In contrast, a high value for the derivative compensates a low price with high demand.

Elasticity and revenue



Elasticity and revenue

The relative change of the revenue induced by a change in prices equals $\dfrac{\Delta R}{\Delta p} = q + p\,\dfrac{\Delta q}{\Delta p}$. This change is positive if
$q + p\,\dfrac{\Delta q}{\Delta p} > 0$ or $-\dfrac{\Delta q}{\Delta p}\,\dfrac{p}{q} < 1.$
$\Rightarrow$ If $\varepsilon_p < 1$, the revenue will increase with an increasing price.

- $\varepsilon_p > 1$: elastic demand
- $\varepsilon_p < 1$: inelastic demand

Elasticity and revenue

- A producer has to evaluate the effect of a price change on the one hand and a quantity change on the other hand
- What is the effect of a price change on demanded quantity and how does it change the revenue?
- The answer is the calculation of the marginal revenue (MR)

From $\Delta R = q\Delta p + p\Delta q$ we receive

$$MR = \frac{\Delta R}{\Delta q} = p(q) + q\,\frac{\Delta p(q)}{\Delta q}$$

Elasticity and marginal revenue

Example

- We take (inverse) demand $p(q) = a - bq$
- the derivative with respect to q is $\frac{\Delta p}{\Delta q} = -b$
- the price elasticity of demand is

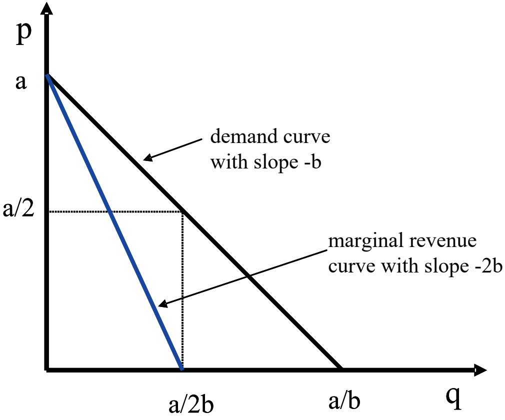
$$\varepsilon_p = (a - bq)/bq$$

- marginal revenue equals

$$MR = \frac{\Delta R}{\Delta q} = p(q) - bq = a - 2bq$$

Elasticity and marginal revenue

$$p(q) = a - bq, \quad MR = a - 2bq \quad \varepsilon_p = (a - bq)/bq$$



Elasticity and marginal revenue

$$p(q) = a - bq, \quad MR = a - 2bq \quad \varepsilon_p = (a - bq)/bq$$

- the MR curve shows twice the slope of the demand curve
- MR is zero for $\varepsilon_p = 1$ and negative for $\varepsilon_p < 1$.
- for $\varepsilon_p < 1$ an increase in produced quantity decreases revenue

Constant elasticity

Demand curves with constant elasticity show a constant effect of price changes on revenue.
The demand function described by

$$D(p) = A\,p^{-\varepsilon}$$

has the constant elasticity ε.

Elasticity

The term “elasticity” is used in different contexts:

- price elasticity of demand
- income elasticity of demand
- cross-price elasticity of demand
- elasticity of supply
- intertemporal elasticity of substitution (of a utility function)

Elasticity – exercise

We are at the market for hamster wheels. The demand for hamster wheels is driven by the demand of two groups of consumers: hamster breeders and hamster owners. Their demand function is as follows:
hamster breeders:

$$q_b(p) = max\left\{200 - p, 0\right\}$$

hamster owners:

$$q_o(p) = max\left\{90 - p, 0\right\}$$

a) Calculate the price elasticity for the demand of hamster breeders and hamster owners for price p.
b) Calculate the price at which the price elasticity of demand of hamster breeders and hamster owners equals 1.

Elasticity – exercise

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hamster owners:

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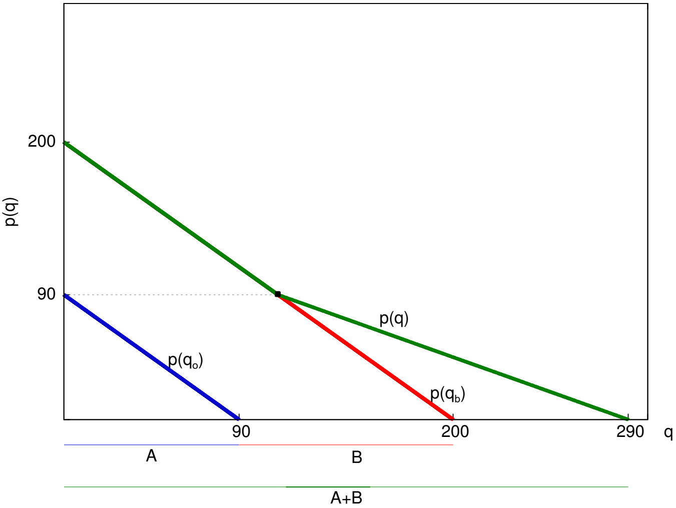
d) Draw the demand curve of hamster breeders and hamster owners. Determine the market demand and draw it.

e) Derive the equation for the market demand of hamster wheels.

f) Calculate the price at which the price elasticity of market demand equals 1.

Exercise – solution



Elasticity – exercise

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f) Which price maximizes the revenue for hamster wheels?
g) Are hamster wheels sold to hamster breeders, hamster owners or both groups of consumers under the revenue-maximizing price?

Theory of production

- description of a company's supply
- consideration of constraints and the objective function of a company; maximizing the objective function.
- the company is essentially restricted by the provided technology which describes the transformation of input to output
- classical input factors are: labor and capital supplemented by land and resources

Production technology

- an output is produced by the use of n input factors: $x = (x_1, \dots, x_n)$

$\rightarrow$ we usually restrain to 2 input factors, x_1 und x_2 (e.g. labor and capital).

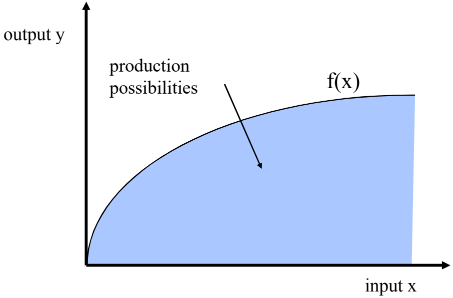
- input factors are used to produce final goods for consumers or intermediate products for other companies: $y = (y_1, \dots, y_m)$

$\rightarrow$ we usually restrain to 1 output y.

- technology describes the transformation of input factors into output: $y = f(x_1, x_2)$ which yields the so-called production function.
- the production function from a formal point of view is very similar to the utility function we already know from the optimal choice theory of consumption

Production function

Production function with only one input factor



- area below the graph: total possibilities of production of the company

$\rightarrow$ determined by the used technology

Input factors

- a company usually uses more than one input factor for production
- is substitution of one input factor by another reasonable, input factors are substitutive.
- is production of the output possible only by input factors in a fixed ratio, the technology is complementary (Leontief production function).
- all combinations of input factors yielding the same output level are described by an isoquant:

$$I_y \;\rightarrow\; y = f(x_1, x_2)$$

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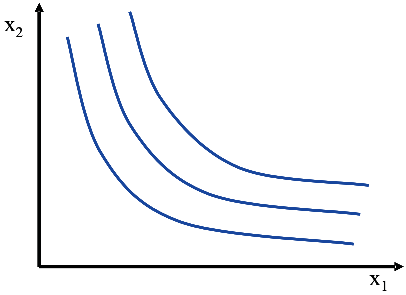
Isoquants of a production function

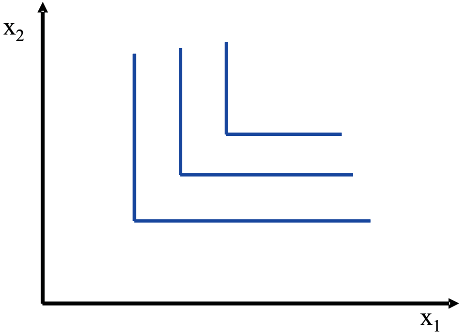


- isoquants are monotonously decreasing

$\Rightarrow$ isoquants further away from the origin reflect a higher production level

Exemplary production technologies

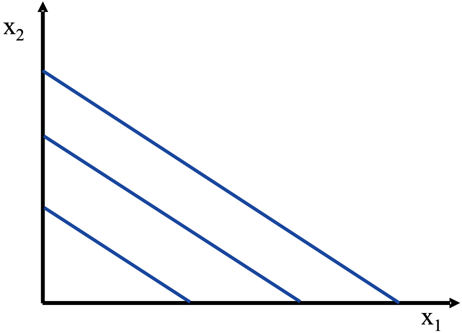
(i) Leontief-technology (complementary)
use of input factors in a fixed ratio $y = \min\{x_1, x_2\}$.



Exemplary production technologies

(ii) Perfect substitutes

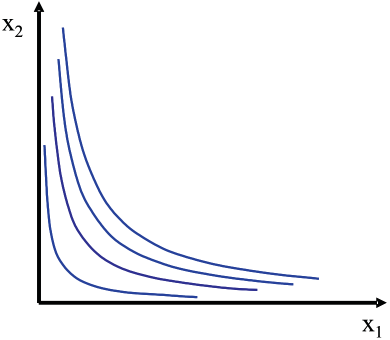
input factors are perfect substitutes $y = x_1 + x_2$.



Exemplary production technologies

(iii) Cobb-Douglas production function

both input factors are necessary for production but partially substitutive: $y = x_1^{\alpha}\, x_2^{\beta}$.



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Marginal product

- reaction of the output on the increase of a production factor

$$\frac{\Delta y}{\Delta x_1} = \frac{f(x_1 + \Delta x_1, x_2) - f(x_1, x_2)}{\Delta x_1}$$

or as differential quotient:

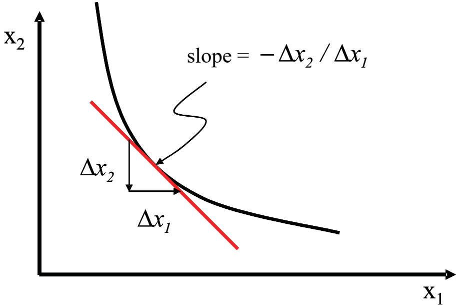
$$\frac{\partial f(x_1, x_2)}{\partial x_1}$$

- the marginal product (MP) is usually positive (exception: Leontief production function)

Technical rate of substitution (TRS)

- exchange ratio of input factors for constant output level

$$TRS(x_1, x_2) = \left|\frac{\Delta x_2}{\Delta x_1}\right| = \frac{MP_1}{MP_2} = \left|\frac{\partial x_2(y, x_1)}{\partial x_1}\right|$$



Economies of scale

The effect of a simultaneous increase of all input factors is measured by the so-called economies of scale

If all input factors are increased by a factor $t > 1$, we face

constant economies of scale if the output equals the original output multiplied with t, $f(tx_1, tx_2) = tf(x_1, x_2)$

increasing economies of scale if the output increases to more than the original output multiplied with t $f(tx_1, tx_2) > tf(x_1, x_2)$

decreasing economies of scale if the output increases to less than the original output multiplied with t $f(tx_1, tx_2) < tf(x_1, x_2)$

Economies of scale

- economies of scale are always a local property

$\Rightarrow$ they can change with the combination of input factors

$\Rightarrow$ a company may face increasing economies of scale followed by constant and finally decreasing economies of scale

Minimizing costs

Which combination of (x_1, x_2) is optimal to produce the quantity y?

cost minimizing combination of input factors

$$\begin{aligned} \min_{x_1,x_2} \quad & w_1x_1 + w_2x_2 \\ \text{considering the constraint:} \quad & y = f(x_1, x_2) \end{aligned}$$

The solution of this problem for any w_1, w_2, y is given by the

cost function or cost curve

$$c(w_1, w_2, y)$$

Minimizing costs

Isocost line

We look for the combination of input factors resulting in identical cost (while assuming constant prices for input factors):

$$\overline{c} = w_1 x_1 + w_2 x_2$$

or

$$x_2 = \frac{\overline{c}}{w_2} - \frac{w_1}{w_2} \cdot x_1$$

The combination of input factors leading to identical cost are located at a line $\rightarrow$ isocost line.

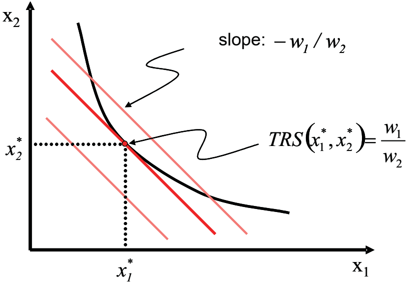
Minimizing costs

optimum condition of minimal cost

For a given output level we are searching for the lowest cost level

$\rightarrow$ tangent point of isocost line and production isoquant

$$TRS(x_1^*, x_2^*) = \frac{w_1}{w_2} = \frac{MP_1(x_1^*, x_2^*)}{MP_2(x_1^*, x_2^*)}$$



Theory of production – exercise

Assume a company produces under the following technology:

$$f(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{3}{2}}$$

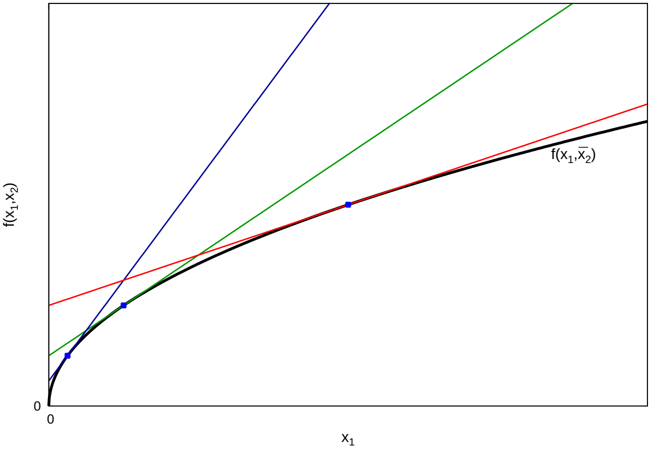
a) Calculate the marginal products (MP).

b) How does MP_1 change with respect to increasing x_1 for constant x_2?

c) How does the company's output change with an increasing x_2 assuming constant x_1? Is the output change equal for every increase in x_2?

d) How does an increase of x_2 affect MP_1?

Exercise – solution



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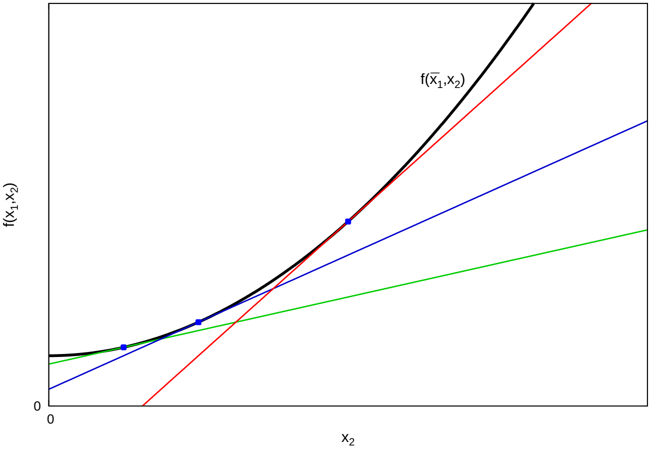

Exercise – solution