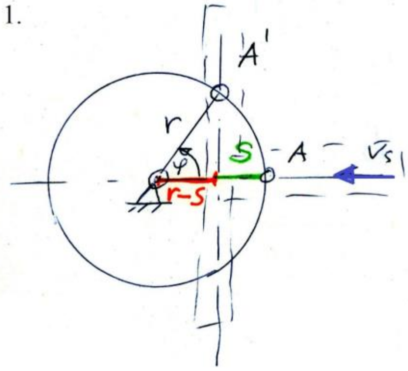
Musterlösung 2. Probeklausur

1.



$$\boxed{s = v_s \cdot t}$$

$$\cos\varphi = \frac{r-s}{r} = 1-\frac{s}{r} = 1-\frac{v_s \cdot t}{r}$$

$$\varphi = \arccos\left(1-\frac{v_s\cdot t}{r}\right)$$

$$\omega = \dot{\varphi}$$

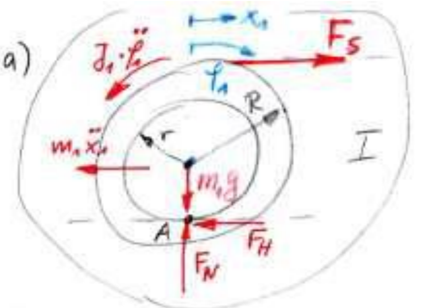
$$\dot{\varphi} = -\frac{1\cdot(-)\frac{v_s}{r}}{\sqrt{1-\left(1-\frac{v_s t}{r}\right)^2}}$$

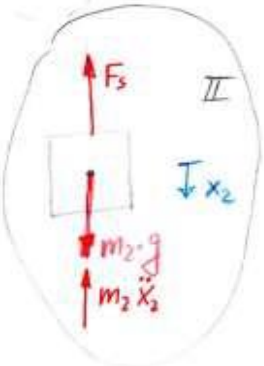
Ableitung

$$\frac{d(\arccos t)}{dt} = -\frac{1}{\sqrt{1-x^2}}$$

$$\omega = \dot{\varphi} = \frac{\frac{v_s}{r}}{\sqrt{1-\left(1-\frac{v_s\cdot t}{r}\right)^2}}$$

2.) a)





b)

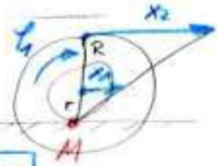
$$I:\ \curvearrowleft A:\ J_1\ddot{\varphi}_1 + m_1\ddot{x}_1\cdot r - F_S\cdot(R+r) = 0$$

$$II:\ \uparrow:\ F_S + m_2\ddot{x}_2 - m_2 g = 0$$

$$F_S = m_2 g - m_2\ddot{x}_2$$

$$J_1\ddot{\varphi}_1 + m_1\ddot{x}_1\cdot r - (m_2\cdot g - m_2\ddot{x}_2)(R+r) = 0$$

Zwangsbed.: $\frac{x_1}{r} = \frac{x_2}{R+r} \Rightarrow x_1 = x_2\cdot\frac{r}{R+r}$



$$\boxed{\ddot{x}_1 = \ddot{x}_2\ \frac{r}{R+r}}$$

Rollbed.: $x_1 = r\cdot\varphi_1$

$\ddot{x}_1 = r\cdot\ddot{\varphi}_1 \Rightarrow$ $\boxed{\ddot{\varphi}_1 = \frac{\ddot{x}_1}{r} = \frac{\ddot{x}_2}{R+r}}$

$J_1 = 4\,m_1 r^2$ (vorgegeben)

$$4m_1r^2\cdot\frac{\ddot{x}_2}{R+r} + m_1\cdot\ddot{x}_2\,\frac{r}{R+r}\cdot r - m_2\,g(R+r) + m_2\ddot{x}_2\,(R+r) = 0$$

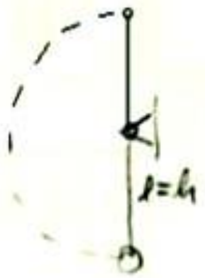
$$\ddot{x}_2\left[\frac{4m_1r^2}{R+r} + \frac{m_1r^2}{R+r} + m_2(R+r)\right] = m_2\,g\,(R+r) \quad \Big|\cdot(R+r)$$

$$\ddot{x}_2\left[5m_1r^2 + m_2\,(R+r)^2\right] = m_2\,g\,(R+r)^2$$

$$\ddot{x}_2 = \frac{m_2\cdot g\cdot(R+r)^2}{\left[5m_1r^2 + m_2\,(R+r)^2\right]} = \frac{10\,kg\cdot 9{,}81\,\frac{m}{s^2}\,(0{,}5)^2\,m^2}{\left[5\cdot 25\,kg\cdot 0{,}2^2\,m^2 + 10\,kg\cdot 0{,}5^2\,m^2\right]}$$

$$\ddot{x}_2 = 3{,}27\,\frac{m}{s^2}$$

3 a)



$$v_1 = \sqrt{2g \cdot 2l} = 4{,}43\ \frac{m}{s}$$

b)

$$m_1 \cdot v_1 = m_1 \cdot u_1 + m_2 \cdot u_2 \qquad (2)$$

$$k = \frac{u_1 - u_2}{-v_1} \Rightarrow u_2 = k \cdot v_1 + u_1 \qquad (1)$$

(2) in (1):

$$m_1 \cdot v_1 = m_1 \cdot u_1 + m_2 \left(k \cdot v_1 + u_1\right)$$

$$m_1 \cdot v_1 = m_1 u_1 + m_2 \cdot k \cdot v_1 + m_2 \cdot u_1$$

$$u_1 (m_1 + m_2) = m_1 v_1 - m_2 \cdot k \cdot v_1$$

$$u_1 = v_1 \cdot \frac{m_1 - k \cdot m_2}{m_1 + m_2} = \left(\frac{0{,}5 - 0{,}9 \cdot 2}{2{,}5}\right) 4{,}43 = -2{,}30\ m/s$$

$$u_2 = 0{,}9 \cdot 4{,}43 - 2{,}3 = 1{,}69\ m/s$$

c)

$$W_{kin,h} = W_{pot}$$

$$\frac{1}{2} m_2 \cdot u_2^2 = m_2 g \cdot h \Rightarrow h = \frac{u_2^2}{2 g} = 0{,}145\ m$$

4. a)

$$m_{ges} = m_{Zyl.} + 3 \cdot m_{Speiche} + m_{Ring}$$

$$m_{Zyl} = \rho \cdot \frac{\pi \cdot 0{,}15^2}{4} \cdot 0{,}15 = 0{,}021\ kg$$

$$m_{Sp.} = 0{,}2 \cdot 0{,}15 \cdot 0{,}08 \cdot \rho = 0{,}019\ kg$$

$$m_{Ring} = \rho \cdot \frac{\pi}{4} \left(0{,}75^2 - 0{,}55^2\right) \cdot 0{,}15 = 0{,}52 - 0{,}28 = 0{,}24\ kg$$

$$m_{ges} = 0{,}021 + 3 \cdot 0{,}019 + 0{,}24 = 0{,}32\ kg$$

b)

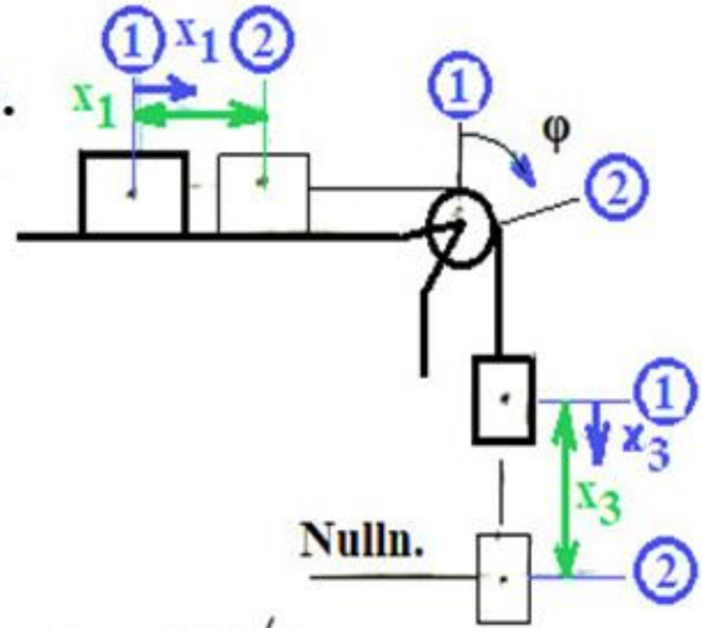
$$J_{ges} = J_{Zyl.} + 3 \left(J_{Sp} + m_{Sp}\, r_{Sp}^2\right) + J_{Ring}$$

$$= \frac{1}{2} m_{Zyl.} \cdot 7{,}5^2 + 3\left(\frac{1}{12} m_{Sp.} 20^2 + m_{Sp.} \cdot 17{,}5^2\right) + \frac{1}{2} \cdot 0{,}52 \cdot 37{,}5^2 - \frac{1}{2} 0{,}28 \cdot 27{,}5^2$$

$$= 0{,}59 + 3(0{,}63 + 5{,}82) + 365{,}63 - 105{,}88\ kg\,mm^2$$

$$J_{ges} = 280\ kg\,mm^2$$

5.



<u>Energiesatz</u>

$$W_{pot(m_3)} - W_{ab} = W_{kin,tr.(m_1)} + W_{kin,rot(m_2)} + W_{kin,tr(m_3)}$$

$$m_3 \cdot g \cdot x_3 - m_1 \cdot g \cdot \mu \cdot x_1 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} J_2 \cdot \omega_2^2 + \frac{1}{2} m_3 \cdot v_3^2$$

<u>Zwangsbed.</u>

$$x_1 = x_3 \; ; \; v_1 = v_3$$

$$v_3 = \omega_2 \cdot r \quad \Rightarrow \omega_2 = \frac{v_3}{r}$$

Massenth. $J_2 = \frac{1}{2} m_2 r_2^2$

$$m_3 \cdot g \cdot x_3 - m_1 \cdot g \cdot \mu \cdot x_3 = \frac{1}{2} m_1 \cdot v_3^2 + \frac{1}{2} m_3 \cdot v_3^2 + \frac{1}{2} m_2 r^2 \cdot \frac{v_3^2}{\cancel{r^2}} \frac{1}{2}$$

$$= v_3^2 \left(\frac{m_1}{2} + \frac{m_3}{2} + \frac{m_2}{4} \right)$$

$$v_3 = \sqrt{\frac{g \cdot x_3 (m_3 - m_1 \cdot \mu)}{\frac{1}{2}(m_1 + \frac{m_2}{2} + m_3)}} = \sqrt{\frac{9{,}81 \cdot 0{,}5\,(5 - 10 \cdot 0{,}15)}{0{,}5\,(16)}}$$

$$\underline{v_3} = \sqrt{\frac{17{,}17}{8}} = \underline{1{,}46 \text{ m/s}}$$