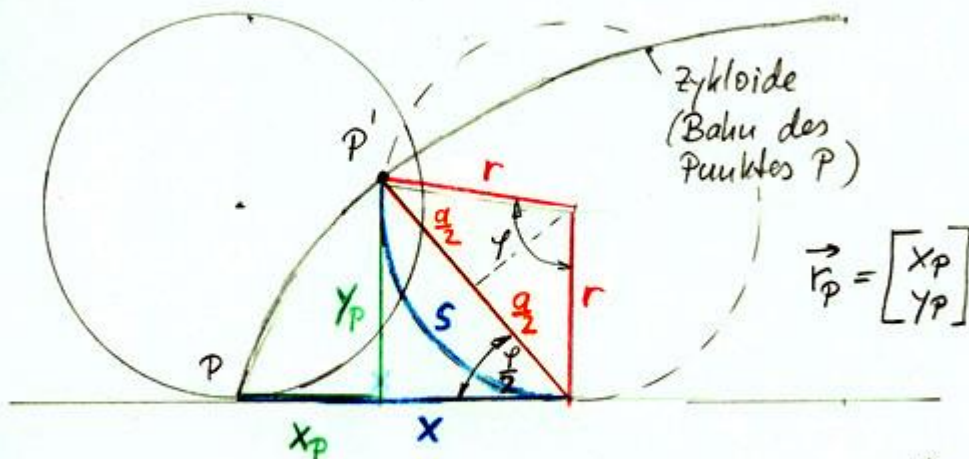


Rollendes Rad: Zyklode (Bewegungsgleichungen)

Rollendes Rad: Gleichung d. Zyklode



$$x = s = r \cdot \varphi \quad ; \quad \frac{a}{2} = r \cdot \sin \frac{\varphi}{2} \Rightarrow a = 2 \cdot r \cdot \sin \frac{\varphi}{2}$$

$$\sin \frac{\varphi}{2} = \frac{y_P}{a} \Rightarrow \boxed{y_P = a \cdot \sin \frac{\varphi}{2} = 2r \cdot \sin^2 \frac{\varphi}{2}}$$

$$(x - x_P)^2 + y_P^2 = a^2$$

$$(x - x_P)^2 = a^2 - y_P^2$$

$$(x - x_P)^2 = (2r \sin \frac{\varphi}{2})^2 - (2r \sin^2 \frac{\varphi}{2})^2$$

$$= (2r \cdot \sin \frac{\varphi}{2})^2 [1 - \sin^2 \frac{\varphi}{2}]$$

$$x - x_P = 2r \cdot \sin \frac{\varphi}{2} \cdot \sqrt{1 - \sin^2 \frac{\varphi}{2}}$$

$$x_P = x - 2r \cdot \sin \frac{\varphi}{2} \cdot \cos \frac{\varphi}{2}$$

$$\underline{x_P = r \cdot \varphi - 2r \cdot \sin \frac{\varphi}{2} \cdot \cos \frac{\varphi}{2}}$$

$$\vec{r}_P = r \cdot \begin{bmatrix} \varphi - 2 \sin \frac{\varphi}{2} \cdot \cos \frac{\varphi}{2} \\ 2 \sin^2 \frac{\varphi}{2} \end{bmatrix} \quad \text{mit } \varphi = \omega \cdot t$$

$$\vec{v}_P = \begin{bmatrix} \dot{x}_P \\ \dot{y}_P \end{bmatrix} = \omega \cdot r \begin{bmatrix} 1 + (\sin^2 \frac{\omega t}{2} - \cos^2 \frac{\omega t}{2}) \\ 2 \cdot \sin \frac{\omega t}{2} \cdot \cos \frac{\omega t}{2} \end{bmatrix}$$

Kontrolle

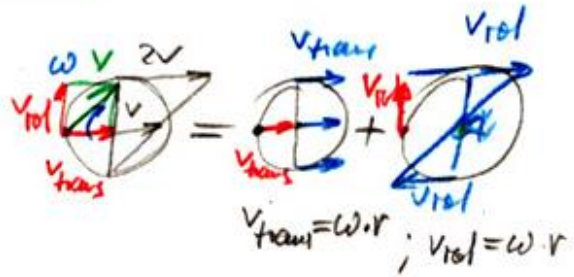
V aus Trans + Rot.

$$\vec{V}_{\text{ges}} = \vec{V}_{\text{trans}} + \vec{V}_{\text{rot}}$$

$$V_{\text{ges}} = \omega \cdot r + \omega \cdot r$$

$$V_{\text{ges}} = 30 \frac{\text{mm}}{\text{s}} + 30 \frac{\text{mm}}{\text{s}}$$

$$V_{\text{ges}} = 42,4 \frac{\text{mm}}{\text{s}} \quad \text{unter } 45^\circ$$



Kontrolle

$$r = 30 \text{ mm}$$

$$\varphi = 90^\circ$$

$$\frac{\varphi}{2} = 45^\circ$$

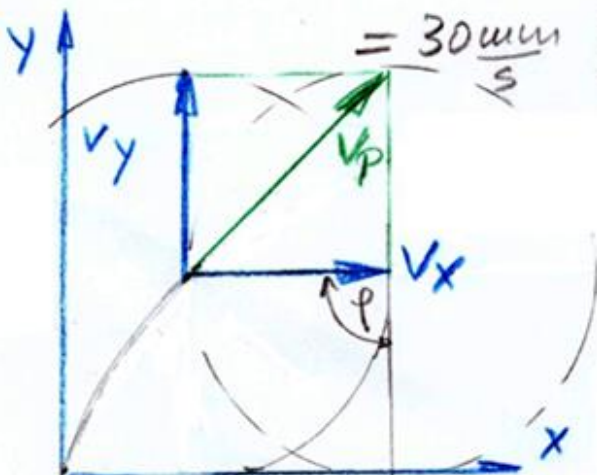
$$\vec{r} = \begin{bmatrix} 17,1 \text{ mm} \\ 30 \text{ mm} \end{bmatrix}$$

Geschw mit $\omega = 1 \frac{1}{\text{s}}$

$$V_P = \begin{bmatrix} \dot{x}_P \\ \dot{y}_P \end{bmatrix} = 30 \frac{\text{mm}}{\text{s}} \begin{bmatrix} 1 + (\sin^2 45^\circ - \cos^2 45^\circ) \\ 2 \cdot \sin 45^\circ \cdot \cos 45^\circ \end{bmatrix}$$

$$= 30 \frac{\text{mm}}{\text{s}} \begin{bmatrix} 1 + 0 \\ 2 \cdot 0,5 \end{bmatrix}$$

$$= 30 \frac{\text{mm}}{\text{s}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 30 \frac{\text{mm}}{\text{s}} \\ 30 \frac{\text{mm}}{\text{s}} \end{bmatrix}$$



$$V_{\text{ges}} = (30^2 + 30^2)^{0,5}$$

$$V_{\text{ges}} = 42,4 \text{ mm/s}$$

$$\vec{a}_p = \begin{bmatrix} \ddot{x}_p \\ \ddot{y}_p \end{bmatrix}$$

$$\ddot{x}_p = a_x = \omega \cdot r \left[\cancel{2 \cdot \sin \frac{\omega t}{2} \cdot \cos \frac{\omega t}{2} \cdot \frac{\omega}{2}} + \cancel{2 \cos \frac{\omega t}{2} \cdot \sin \frac{\omega t}{2} \cdot \frac{\omega}{2}} \right]$$

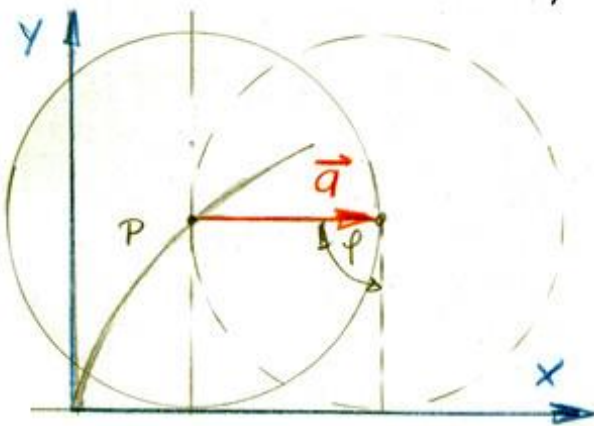
$$\ddot{x}_p = \omega^2 \cdot r \cdot 2 \cdot \sin \frac{\omega t}{2} \cdot \cos \frac{\omega t}{2}$$

$$\ddot{y}_p = a_y = \omega \cdot r \cdot \cancel{2} \left[\cos \frac{\omega t}{2} \cdot \frac{\omega}{2} \cdot \cos \frac{\omega t}{2} + \sin \frac{\omega t}{2} \cdot \cancel{(-\sin \frac{\omega t}{2})} \cdot \frac{\omega}{2} \right]$$

$$\ddot{y}_p = \omega^2 \cdot r \left[\cos^2 \frac{\omega t}{2} - \sin^2 \frac{\omega t}{2} \right]$$

$$\vec{a} = \begin{bmatrix} \ddot{x}_p \\ \ddot{y}_p \end{bmatrix} = \omega^2 \cdot r \begin{bmatrix} 2 \sin \frac{\omega t}{2} \cdot \cos \frac{\omega t}{2} \\ \cos^2 \frac{\omega t}{2} - \sin^2 \frac{\omega t}{2} \end{bmatrix}$$

Kontrolle $r = 30 \text{ mm}$; $\varphi = 90^\circ$; $\frac{f}{2} = 45^\circ$; $\omega = 1 \frac{1}{s}$



$$\vec{a} = 30 \frac{\text{mm}}{\text{s}^2} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} 30 \frac{\text{mm}}{\text{s}^2} \\ 0 \end{bmatrix}$$

Für $\varphi = 180^\circ$; $\varphi_2 = 90^\circ$



$$v_p = 30 \frac{\text{mm}}{\text{s}} \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 60 \frac{\text{mm}}{\text{s}} \\ 0 \end{bmatrix}$$

$$a_p = 30 \frac{\text{mm}}{\text{s}^2} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -30 \frac{\text{mm}}{\text{s}^2} \end{bmatrix}$$

Beschl. aus Trans + Rot

$$\vec{a}_p = \vec{a}_{n,trans} + \vec{a}_{t,trans} + \vec{a}_{n,rot} + \vec{a}_{t,rot}$$

$$\vec{a}_p = 0 + 0 + \frac{v^2}{r} + 0$$



Gesamt = Trans + Rot