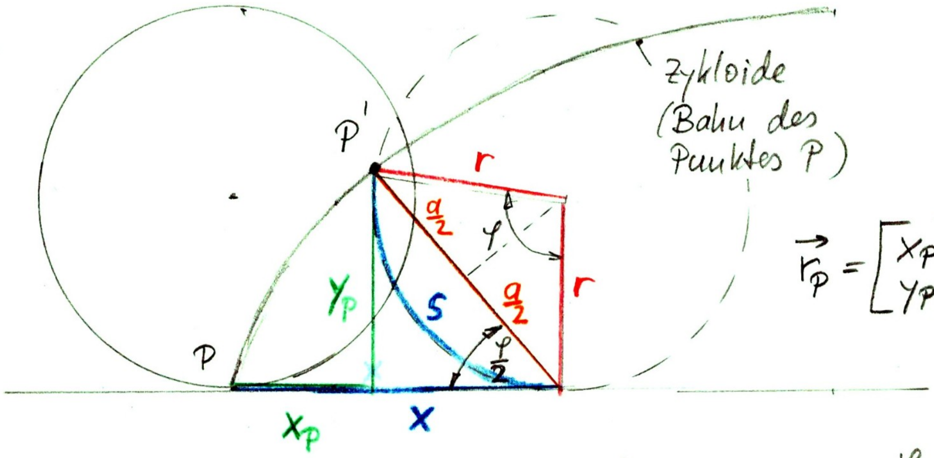
Rollendes Rad: Zykloide (Bewegungsgleichngen)

Rollendes Rad: Gleichung d. Zykloide



$$\vec{r}_P = \begin{bmatrix} x_P \\ y_P \end{bmatrix}$$

$$x = s = r \cdot \varphi \quad ; \quad \frac{a}{2} = r \cdot \sin\frac{\varphi}{2} \Rightarrow a = 2 \cdot r \cdot \sin\frac{\varphi}{2}$$

$$\sin\frac{\varphi}{2} = \frac{y_P}{a} \Rightarrow \boxed{y_P = a \cdot \sin\frac{\varphi}{2} = 2r \cdot \sin^2\frac{\varphi}{2}}$$

$$(x - x_P)^2 + y_P^2 = a^2$$

$$(x - x_P)^2 = a^2 - y_P^2$$

$$(x - x_P)^2 = \left(2r\sin\frac{\varphi}{2}\right)^2 - \left(2r\sin^2\frac{\varphi}{2}\right)^2$$

$$= \left(2r \cdot \sin\frac{\varphi}{2}\right)^2 \left[1 - \sin^2\frac{\varphi}{2}\right]$$

$$x - x_P = 2r \cdot \sin\frac{\varphi}{2} \cdot \sqrt{1 - \sin^2\frac{\varphi}{2}}$$

$$x_P = x - 2r \cdot \sin\frac{\varphi}{2} \cdot \cos\frac{\varphi}{2}$$

$$x_P = r \cdot \varphi - 2r \cdot \sin\frac{\varphi}{2} \cdot \cos\frac{\varphi}{2}$$

$$\vec{r}_P = \vec{r} \begin{bmatrix} \varphi - 2\sin\frac{\varphi}{2} \cdot \cos\frac{\varphi}{2} \\ 2\sin^2\frac{\varphi}{2} \end{bmatrix} \quad \text{mit } \varphi = \omega \cdot t$$

$$\vec{v}_P = \begin{bmatrix} \dot{x}_P \\ \dot{y}_P \end{bmatrix} = \omega \cdot r \begin{bmatrix} 1 + \left(\sin^2\frac{\omega t}{2} - \cos^2\frac{\omega t}{2}\right) \\ 2 \cdot \sin\frac{\omega t}{2} \cdot \cos\frac{\omega t}{2} \end{bmatrix}$$

Kontrolle

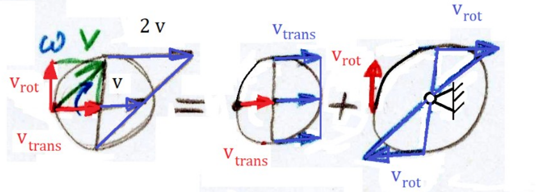
v aus Trans + Rot.

$$\vec{v}_{ges} = \vec{v}_{trans} + \vec{v}_{rot}$$

$$v_{ges} = \vec{\omega \cdot r} + \vec{\omega \cdot r}$$

$$\vec{v}_{ges} = \overrightarrow{30\,\frac{mm}{s}} + \overrightarrow{30\,\frac{mm}{s}}$$

$$v_{ges} = 42{,}4\,\frac{mm}{s} \quad \text{unter } 45^\circ$$



$$v_{trans} = \omega \cdot r \; ; \; v_{rot} = \omega \cdot r$$

Kontrolle

$r = 30\,mm$

$\varphi = 90^\circ$

$\frac{\varphi}{2} = 45^\circ$

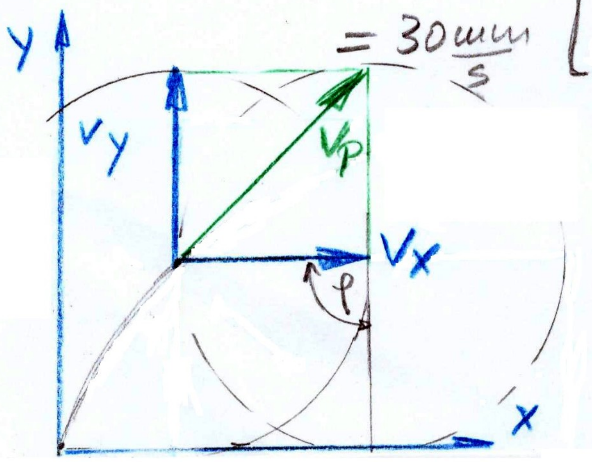
$$\vec{r} = \begin{bmatrix} 17{,}1\,mm \\ 30\,mm \end{bmatrix}$$

Geschw. mit $\omega = 1\,\frac{1}{s}$

$$v_P = \begin{bmatrix} \dot{x}_P \\ \dot{y}_P \end{bmatrix} = 30\,\frac{mm}{s} \begin{bmatrix} 1 + (\sin^2 45^\circ - \cos^2 45^\circ) \\ 2 \cdot \sin 45^\circ \cdot \cos 45^\circ \end{bmatrix}$$

$$= 30\,\frac{mm}{s} \begin{bmatrix} 1 + \quad 0 \\ 2 \cdot \quad 0{,}5 \end{bmatrix}$$

$$= 30\,\frac{mm}{s} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 30\,\frac{mm}{s} \\ 30\,\frac{mm}{s} \end{bmatrix}$$



$$\vec{a}_p = \begin{bmatrix} \ddot{x}_p \\ \ddot{y}_p \end{bmatrix}$$

$$\ddot{x}_p = a_x = \omega \cdot r \left[\cancel{2} \cdot \sin\frac{\omega t}{2} \cdot \cos\frac{\omega t}{2} \cdot \frac{\omega}{\cancel{2}} + \cancel{2} \cos\frac{\omega t}{2} \cdot \sin\frac{\omega t}{2} \cdot \frac{\omega}{\cancel{2}} \right]$$

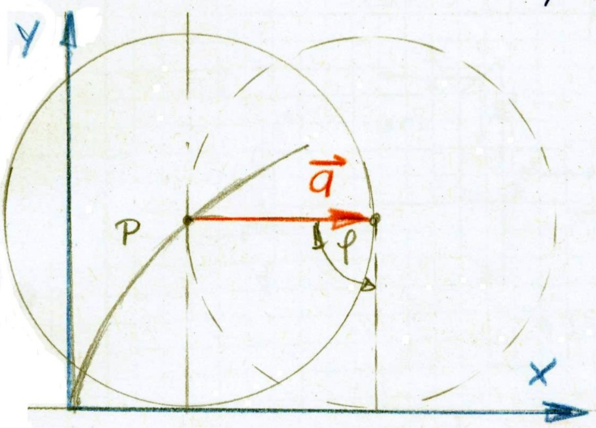
$$\ddot{x}_p = \omega^2 \cdot r \cdot 2 \cdot \sin\frac{\omega t}{2} \cdot \cos\frac{\omega t}{2}$$

$$\ddot{y}_p = a_y = \omega \cdot r \cdot \cancel{2} \left[\cos\frac{\omega t}{2} \cdot \frac{\omega}{\cancel{2}} \cdot \cos\frac{\omega t}{2} + \sin\frac{\omega t}{2} \left(-\sin\frac{\omega t}{2}\right) \cdot \frac{\omega}{\cancel{2}} \right]$$

$$\ddot{y}_p = \omega^2 \cdot r \left[\cos^2\frac{\omega t}{2} - \sin^2\frac{\omega t}{2} \right]$$

$$\vec{a} = \begin{bmatrix} \ddot{x}_p \\ \ddot{y}_p \end{bmatrix} = \omega^2 \cdot r \begin{bmatrix} 2 \sin\frac{\omega t}{2} \cdot \cos\frac{\omega t}{2} \\ \cos^2\frac{\omega t}{2} - \sin^2\frac{\omega t}{2} \end{bmatrix}$$

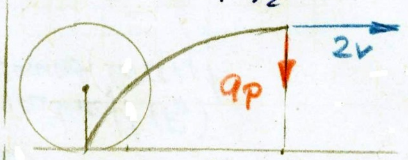
Kontrolle $r = 30\,mm$; $\varphi = 90°$; $\frac{\varphi}{2} = 45°$; $\omega = 1\,\frac{1}{s}$



$$\vec{a} = 30\,\frac{mm}{s^2} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} 30\,\frac{mm}{s^2} \\ 0 \end{bmatrix}$$

Für $\varphi = 180°$; $\frac{\varphi}{2} = 90°$



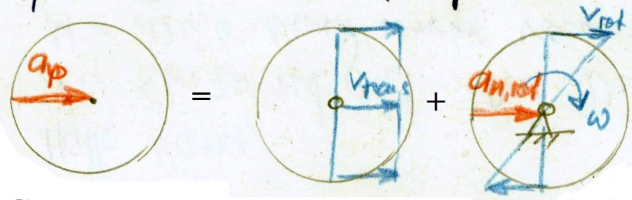
$$V_p = 30\,\frac{mm}{s} \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 60\,\frac{mm}{s} \\ 0 \end{bmatrix}$$

$$a_p = 30\,\frac{mm}{s^2} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -30\,\frac{mm}{s^2} \end{bmatrix}$$

Beschl. aus Trans + Rot

$$\vec{a}_p = \vec{a}_{n,trans} + \vec{a}_{t,trans} + \vec{a}_{n,rot} + \vec{a}_{t,rot}$$

$$\vec{a}_p = 0 + 0 + \frac{v^2}{r} + 0$$



Gesamt = Trans + Rot