Aufgabe 3-4 (Energie / Arbeit)

Schiebende Arbeit

$F = 10\,kN$

$s = 40\,m$

$\alpha = 20^\circ$

$$\boxed{W = F \cdot s}$$

$$W = F \cdot \cos\alpha \cdot s$$

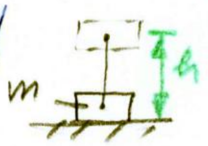
$$W = 10.000\,N \cdot \cos 20^\circ \cdot 40\,m$$

$$W = 375.877\,Nm$$

$$W = 376\,kNm$$

Drehende Arbeit : $\boxed{W_{An} = M \cdot \varphi}$; φ im Bogenmaß

Potentielle Arbeit

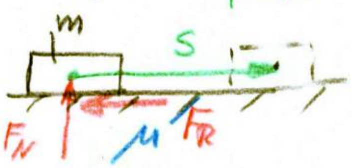


$$\boxed{W_{pot} = m \cdot g \cdot h}$$

Federarbeit :

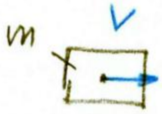
$$\boxed{W_{Feder} = \frac{1}{2} c\, s^2}$$

Reibarbeit :



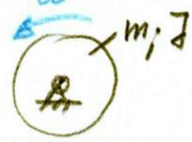
$$\boxed{W_{Reibung} = F_N \cdot \mu \cdot s}$$

Kinetische Translatorische Energie :



$$\boxed{W_{kin,trans} = \frac{1}{2} m v^2}$$

Kinetische Rotations Energie :



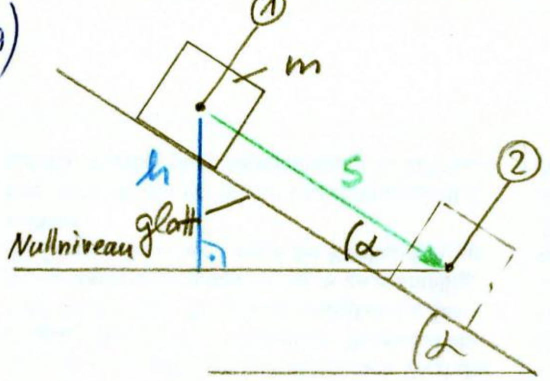
$$\boxed{W_{kin,rot} = \frac{1}{2} J \cdot \omega^2}$$

Energiesatz

$$\sum W_{①} + W_{An} - W_{Reibung} = \sum W_{②}$$

Kleine Aufgaben zu Energiesatz

a)



Die Masse m wird aus der Ruhelage losgelassen. Wie groß ist v im Zustand 2.

Geg: $m = 10\,kg$
$s = 3\,m$
$\alpha = 35°$

$$\sum W_{①} = \sum W_{②}$$

$$W_{pot} = W_{kin,trans}$$

$$\cancel{m} \cdot g \cdot h = \frac{1}{2} \cancel{m} v^2 \Rightarrow v = \sqrt{2g \cdot h} \quad ; \quad h = s \cdot \sin\alpha$$

$$v = \sqrt{2 \cdot 9{,}81 \frac{m}{s^2} \cdot 3m \cdot \sin 35°} = \underline{5{,}81 \frac{m}{s}}$$

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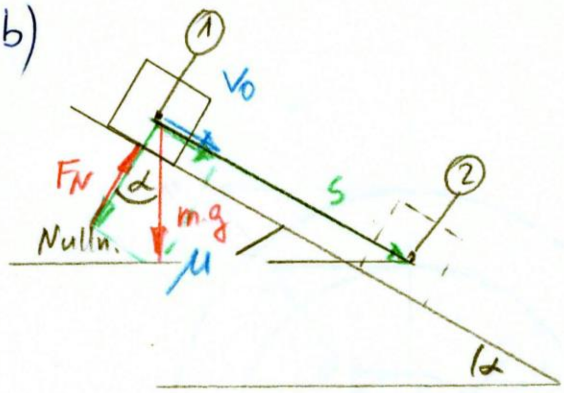
b)



Jetzt mit Vorgeschw. $v_0 = 2 \frac{m}{s}$ und Reibung $\mu = 0{,}2$

$$W_{kin,trans①} + W_{pot} - W_{Reib} = W_{kin,tr②}$$

$$\frac{1}{2} m \cdot v_0^2 + m \cdot g \cdot h - F_N \mu \cdot s = \frac{1}{2} m v^2$$

$h = s \cdot \sin\alpha$
$F_N = m \cdot g \cdot \cos\alpha$

$$\frac{1}{2} \cancel{m} v_0^2 + \cancel{m} \cdot g \cdot s \cdot \sin\alpha - \cancel{m} \cdot g \cdot \cos\alpha \cdot \mu \cdot s = \frac{1}{2} \cancel{m} v^2$$

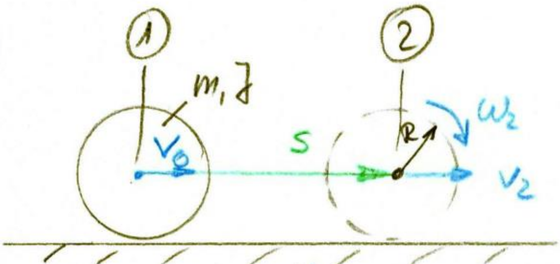
$$v^2 = v_0^2 + 2 \cdot g \cdot s \cdot \sin\alpha - 2 g s \cdot \mu \cdot \cos\alpha$$

$$v^2 = v_0^2 + 2 g \cdot s \, (\sin\alpha - \mu \cos\alpha)$$

$$v = \sqrt{2^2 \frac{m^2}{s^2} + 2 \cdot 9{,}81 \frac{m}{s^2} \cdot 3m \, (\sin 35° - 0{,}2 \cdot \cos 35°)}$$

$$\underline{v = 5{,}3 \frac{m}{s}}$$

c)



$J = \frac{1}{2} m R^2$

$s = 3\,m$; $v_0 = 2\,\frac{m}{s}$

$R = 0{,}5\,m$

Ges: $v_{②}$, ω_2

reines Rollen
ohne Reibung

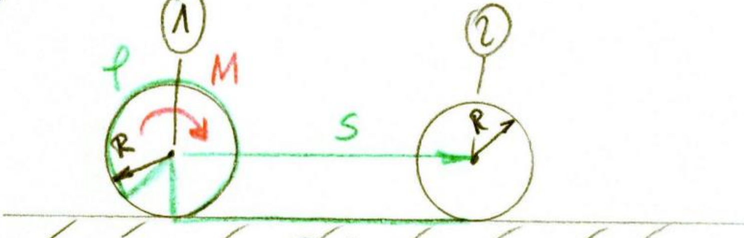
$$W_{kin,trans①} + W_{kin,rot①} = W_{kin,trans②} + W_{kin,rot②}$$

$$\frac{1}{2} m v_0^2 + \cancel{\frac{1}{2} J \omega_0^2} = \frac{1}{2} m v_{②}^2 + \cancel{\frac{1}{2} J \cdot \omega_{②}^2}$$

$$\boxed{v_0 = v_2}$$

Rollbed. $v_2 = \omega_2 \cdot R \Rightarrow \boxed{\omega_2 = \frac{v_2}{R}}$

d)



Geg: $M = 1\,Nm$
$s = 3\,m$
$R = 0{,}5\,m$
$J = \frac{1}{2} m R^2$
$m = 10\,kg$

Ges: v_2, ω_2

reines Rollen
ohne Reibung

$$W_{An} = W_{kin,trans} + W_{kin,rot}$$

$$M \cdot \varphi = \frac{1}{2} m v_2^2 + \frac{1}{2} J \omega_2^2$$

Rollbed: $\boxed{s = R \cdot \varphi}$; $\boxed{v_2 = R \cdot \omega_2}$

$$M \cdot \frac{s}{R} = \frac{1}{2} m v_2^2 + \frac{1}{2}\frac{1}{2} m \cancel{R^2} \cdot \left(\frac{v_2}{\cancel{R}}\right)^2$$

$$M \frac{s}{R} = \frac{1}{2} m v_2^2 + \frac{1}{4} m v_2^2$$

$$M \frac{s}{R} = v_2^2 (0{,}75\,m) \Rightarrow v_2 = \sqrt{\frac{M \cdot s}{R \cdot 0{,}75\,m}}$$

$$\underline{v_2} = \sqrt{\frac{1\,Nm \cdot 3\,\cancel{m}}{0{,}5\,\cancel{m} \cdot 0{,}75 \cdot 10\,kg}} = \underline{0{,}89\,\frac{m}{s}}$$

$$\underline{\omega_2} = \frac{v_2}{R} = \frac{0{,}89}{0{,}5}\,\frac{1}{s} = \underline{1{,}79\,\frac{1}{s}}$$