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Energy Economics

Fachbereich 2 Informatik und Ingenieurwissenschaften

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Preferences

What do preferences describe?

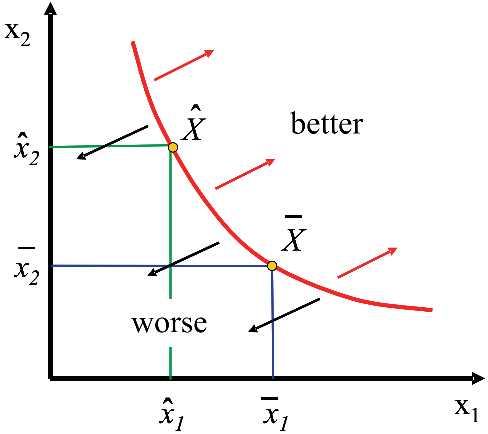
- Consumers can choose between *bundles of goods*.
- The quantities within a bundle of goods *completely* describe all relevant alternatives.
- preferences correspond to the consumer's relative appreciation for possible alternatives.

preferences allow to answer e.g. the question

2 wine and half a pizza or

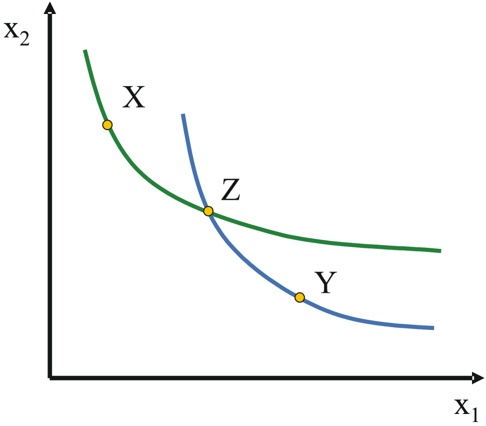
1 wine and a complete pizza?

Indifference curve



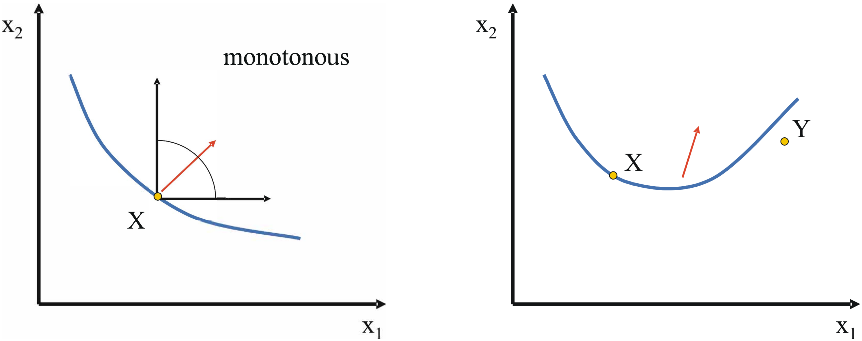
Consumers are indifferent between all bundles of goods located on the indifference curve, e.g. $\bar{X} \sim \hat{X}$.

Several indifference curves



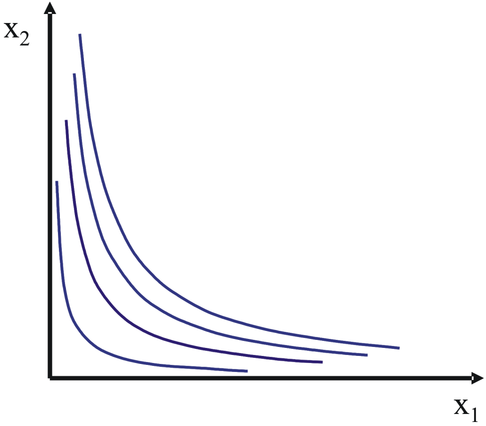
Two indifference curves can never intersect.

Monotony



When Y is higher in both quantities it is preferred to X. This only applies for a monotonously decreasing slope like in the left graph.

Usual preferences



Preferences are monotonous and convex.

Utility and preferences

- every bundle of goods is assigned with a number
- the number allows a comparison of different bundles
- a bundle will be assigned with a higher number if the consumer prefers this bundle over another
- $X \succ Y \quad \Longleftrightarrow \quad u(X) > u(Y)$
- The utility function $u(\cdot)$ reflects the consumer's preferences

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Utility function

Example:
preferences shall be $A \succ B \succ C$.
$\Rightarrow$ there is an infinite number of utility functions describing this situation

$$\begin{array}{lll} u_1(A) = 2 & u_2(A) = 8 & u_3(A) = 0 \\ u_1(B) = 1 & u_2(B) = 4 & u_3(B) = -1 \\ u_1(C) = 0 & u_2(C) = 0 & u_3(C) = -24 \end{array}$$

An exemplary utility function:

$$u(x_1, x_2) = x_1 \cdot x_2$$

Ordinality and cardinality

ordinality

So far we use and work with the ordinality of preferences, e.g.:

- X is at least as good as Y

cardinality

In contrast, the cardinality of a utility function allows a nominal valuation of bundles of goods which expands the ordinal concept of preferences.

- The utility of X corresponds the real number $U(X)$
- A consumer prefers X over Y if $U(X) > U(Y)$

Ordinality and cardinality

The cardinality of utility functions allows the following comparisons:

- The utility of consumer A from the bundle X is higher than the utility of consumer B from Y.
- The sum of the consumers' utilities from X and Y is higher than the sum of their utilities X' and Y'.

Ordinality requires less assumptions and is consistent for consumption theory. Nevertheless, the cardinality of utility functions is easy to handle and thus also has advantages.

Indifference curves of a utility function

Let us use the utility function $u(x_1, x_2) = x_1 \cdot x_2$ in the following as an example.

Calculation of indifference curves

We know that all bundles (x_1, x_2) located on the same indifference curve result in the same utility: $u(x_1, x_2) = x_1 \cdot x_2 = \bar{u}$. The indifference curve can be seen as a function depending on x_1

$$x_2(x_1) = \bar{u}/x_1$$

Notation:

For the indifference curve of the utility level $\bar{u}$ we write

$$I_{\bar{u}}$$

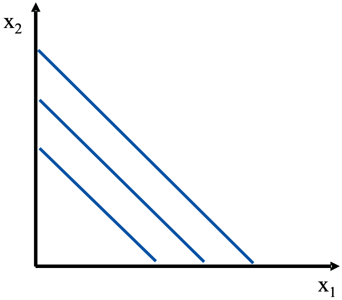
.

Examples for different utility functions

(i) Perfect substitutes

How do the indifference curves of the utility function $u(x_1, x_2) = x_1 + x_2$ look like?

$$x_2 = \bar{u} - x_1$$

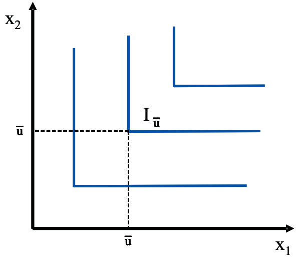


Examples for different utility functions

(ii) Perfect complements

The utility function $u(x_1, x_2) = \min\{x_1, x_2\}$ has indifference curves

$$x_2 \begin{cases} \text{not defined for } x_1 < \bar{u} \\ \geq \bar{u}, \text{ for } x_1 = \bar{u} \\ = \bar{u}, \text{ for } x_1 > \bar{u} \end{cases}$$

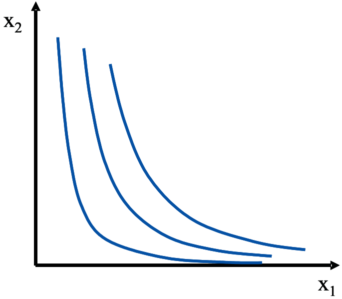


Examples for different utility functions

(iii) Cobb-Douglas utility function

How do the indifference curves of the utility function $u(x_1, x_2) = x_1^{\alpha} \cdot x_2^{\beta}$ look like?

$$x_2 = (\bar{u}/x_1^{\alpha})^{\frac{1}{\beta}}$$



Marginal utility

Marginal utility (MU) indicates the marginal change of utility for a marginal change in the quantities the bundle of goods consists of.

$$MU_1 := \frac{\Delta u}{\Delta x_1} = \frac{u(x_1 + \Delta x_1, x_2) - u(x_1, x_2)}{\Delta x_1}$$

$$MU_2 := \frac{\Delta u}{\Delta x_2} = \frac{u(x_1, x_2 + \Delta x_2) - u(x_1, x_2)}{\Delta x_2}$$

The total change of utility can be approximately expressed by

$$\Delta u = MU_1 \Delta x_1 + MU_2 \Delta x_2$$

Marginal utility II

In a continuously differentiable function **marginal utility** is simply the derivative of the utility function with respect to the quantity of goods.

$$MU_1 := \frac{\partial u}{\partial x_1}$$

$$MU_2 := \frac{\partial u}{\partial x_2}$$

The total change of utility can be expressed by the total differential

$$\mathrm{d}u = MU_1 \mathrm{d}x_1 + MU_2 \mathrm{d}x_2$$

Marginal rate of substitution

- Marginal utility indicates the impact of the change of one good on utility
- **Marginal rate of substitution (MRS)** indicates the consumer's willingness to exchange one good for another

indifference

For indifference we find: $\Delta u = MU_1 \Delta x_1 + MU_2 \Delta x_2 = 0$ or

$$\frac{MU_1}{MU_2} = -\frac{\Delta x_2}{\Delta x_1} := MRS$$

MRS corresponds to the inverted ratio of MUs

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Marginal rate of substitution II

We can use the total differential

$$\mathrm{d}u = MU_1 \mathrm{d}x_1 + MU_2 \mathrm{d}x_2 = 0$$

instead of marginal differences

$$\Delta u = MU_1 \Delta x_1 + MU_2 \Delta x_2 = 0$$

to express **indifference** if the function is totally differentiable. Thus, for a totally differentiable function we receive

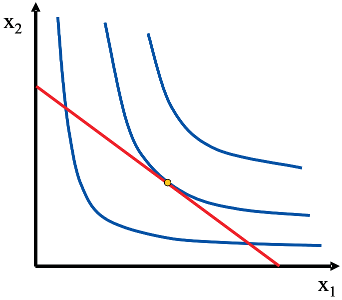
$$\frac{MU_1}{MU_2} = -\frac{\mathrm{d}x_2}{\mathrm{d}x_1} := MRS$$

instead of

$$\frac{MU_1}{MU_2} = -\frac{\Delta x_2}{\Delta x_1} := MRS$$

Optimal consumption decision

The best bundle of goods within the budget



The budget line and the indifference curve determine the optimal decision of the consumer,
the "best" what is "affordable"!

Optimal consumption decision

- In the optimum there is no intersection between **indifference curve** and **budget line**
- Usually the optimum is determined by the tangent point of indifference curve and budget line

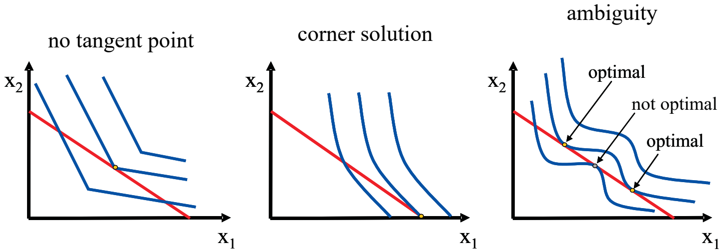
  In this case the optimum is defined as:

$$-\frac{\Delta x_2}{\Delta x_1} = \frac{p_1}{p_2} = MRS$$

We call the described optimum condition the marginal condition of the optimal consumer's decision.

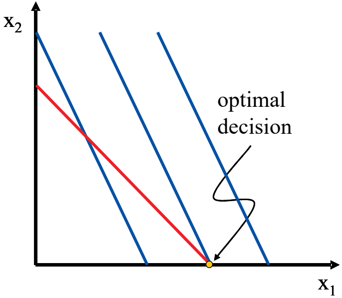
Exceptions

The marginal condition does not determine the optimum in every case



Examples for optimal decisions

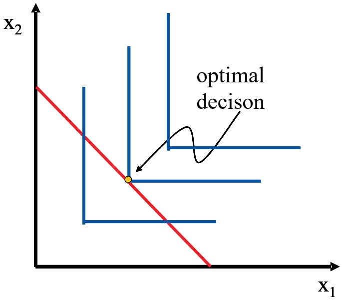
(i) Optimal decision facing perfect substitutes



Facing perfect substitutes the consumer usually consumes only one good.

Examples for optimal decisions

(ii) Optimal decision facing perfect complements



Facing perfect complements the consumer consumes both goods in a certain ratio.

Examples for optimal decisions

(iii) Optimal decision facing Cobb-Douglas-preferences

Cobb-Douglas utility functions have several advantageous properties

In the following we use the utility function

$$u(x_1, x_2) = x_1^{\alpha} \cdot x_2^{\beta}$$

with the budget line $p_1 x_1 + p_2 x_2 = m$.

Hence, marginal utility is

$$MU_1 = \alpha x_1^{(\alpha-1)} x_2^{\beta} \quad \text{und} \quad MU_2 = \beta x_1^{\alpha} x_2^{\beta-1}$$

and the marginal rate of substitution

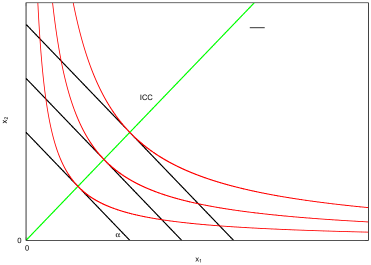
$$MRS = \frac{MU_1}{MU_2} = \frac{p_1}{p_2} \quad \text{thus yields} \quad \frac{p_1}{p_2} = \frac{\alpha}{\beta}\frac{x_2}{x_1}$$

Income Consumption Curve

Rearrangement of the last equation with respect to x_2 yields the income consumption curve (ICC)

$$x_2 = \frac{p_1}{p_2}\frac{\beta}{\alpha}x_1$$

The ICC contains all optimal bundles along an increasing income path.



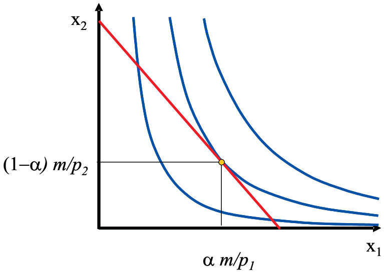
Cobb-Douglas preferences

Using the budget line yields

$$x_1 = \frac{\alpha}{\alpha + \beta} \frac{m}{p_1} \text{ and } x_2 = \frac{\beta}{\alpha + \beta} \frac{m}{p_2}$$

Using $\beta = 1 - \alpha$ (constant economies of scale) yields

$$x_1 = \alpha \frac{m}{p_1} \text{ und } x_2 = (1 - \alpha) \frac{m}{p_2}$$



Demand curve

The optimal consumption quantities x_1 and x_2 correspond to demand $x_1(p_1)$, $x_2(p_2)$.
However, in economics usually inverse demand is used for illustration which is the result of an easy rearrangement of demand with respect to prices

$$p_1 = \frac{\alpha}{\alpha + \beta} \frac{m}{x_1}$$

and

$$p_2 = \frac{\beta}{\alpha + \beta} \frac{m}{x_2}$$

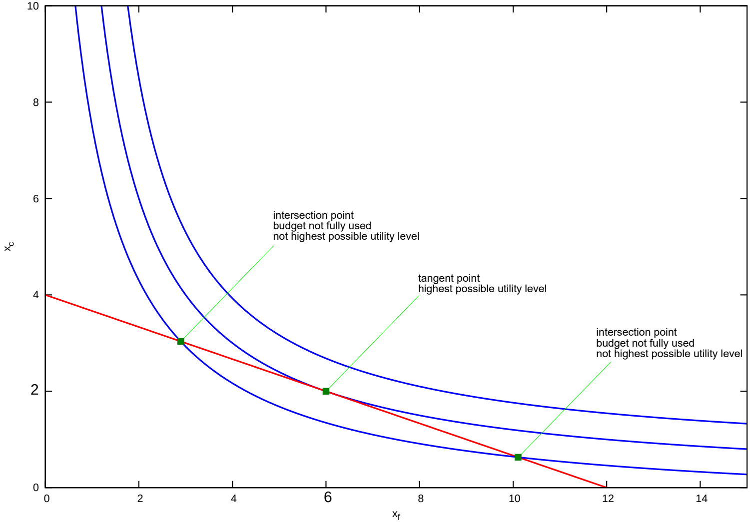
For simplicity demand is often assumed to be linear

Exercise

Carl can spend 48 € for food and clothes each week. The utility from these goods is approximately described by the following equation $u(x_f, x_c) = x_f x_c$.
Assume that food costs 4 € per unit while the unit price for clothes is 12 €

- Draw Carl's budget line
- Calculate the MRS for the utility maximum
- Calculate the optimal combination of food and clothes. Illustrate your result

Exercise – solution



Demand

Aggregation
The aggregation of individual demand yields the aggregated demand at a market.

Since we are interested in the demand for one good, we assume all other prices and income as exogenously given.

$\longrightarrow$ partial analysis (*ceteris paribus*)

- neglects the substitution effect on the markets of other goods.
- Examination of these effects requires a total analysis, e.g. general equilibrium model

Aggregation

In the following we focus on one good (x_1 or x_2)

Individual demand

The individual utility function $u^i(x_1, x_2)$ and the consumer's budget constraint $p_1 x_1 + p_2 x_2 \leq m^i$ determine the demand of the consumer with respect to good 1, $x_1^i(p_1, p_2, m^i)$.

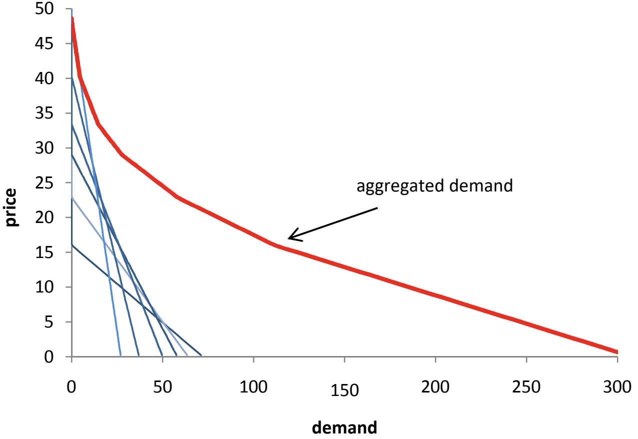
Aggregated demand

The aggregated demand at a market, thus also called markt demand of n consumers is given by

$$X_1(p, m^1, \ldots, m^n) = \sum_{i=1}^{n} x_1^i(p_1, p_2, m^i)$$

Aggregated demand curve

Since individual demand for normal goods decreases with increasing prices, aggregated demand shows the same behavior $D(p) = \sum_{i=1}^{n} D^i(p)$.



Reaction of demand on price changes

- We assume a normal good leading to a decreasing demand with increasing prices
- How strong is the reaction of demand on price changes?
- For comparability we focus on relative changes

In order to measure the “sensitivity” of demand with respect to a price change, we use the (price) elasticity of demand to answer the question:

> What is the change of demand on a percentage basis,
> if the price changes 1 %?

Price elasticity of demand

$$\varepsilon_p = \left| \frac{\text{\% change of demand}}{\text{\% change of prices}} \right| = -\frac{\Delta x / x}{\Delta p / p} = \frac{\Delta x}{\Delta p} \, \frac{p}{x}$$

or for a continuously differentiable demand function

$$\varepsilon_p = -\frac{d\,D(p)}{dp} \cdot \frac{p}{D(p)}$$

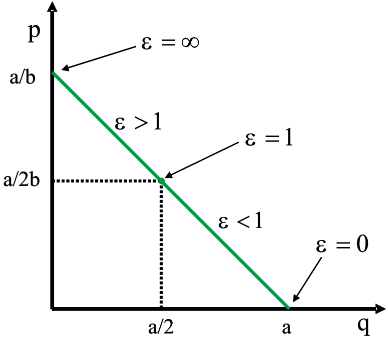
- Since the demand for normal goods decreases with increasing prices, the slope (respectively the derivative $d\,D(p)/dp$) is always negative. By definition this compensated by a minus.

Elasticity of demand along the price curve

- How price elasticity depends on prices?
- Example linear demand curve: $D(p) = a - bp$.
- The slope of the demand curve is $\frac{d\ D(p)}{dp} = -b$
- the elasticity is

$$\varepsilon_p = -\frac{d\ D(p)}{dp} \cdot \frac{p}{D(p)} = b\ \frac{p}{a - bp}$$

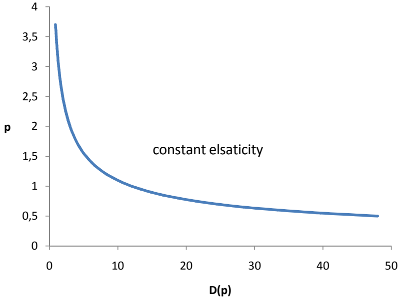
Elasticity of demand along the price curve



The elasticity of demand changes along the demand curve although the slope is constant. It varies between $\varepsilon = 0$ (for $p = 0$) and $\varepsilon = \infty$ (for $D(p) = 0$).

Constant elasticity of demand

Price elasticity along the following curve is constant.



- A high price with low demand is compensated by a small value for the derivative $D'(p)$.
- In contrast, a high value for the derivative compensates a low price with high demand.

Elasticity and revenue

Revenue

Changing prices (usually) affect the producer's revenue which is given by $R = px$.

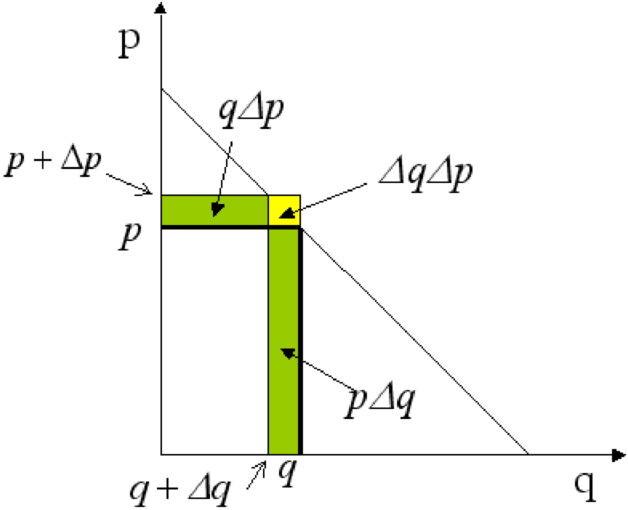
A change in prices and (thus) sold quantity (usually) leads to a changed revenue

$$R + \Delta R = (p + \Delta p)(q + \Delta q) = pq + q\Delta p + p\Delta q + \Delta p \, \Delta q$$

Thus, the change in revenue is $\Delta R = q\Delta p + p\Delta q + \Delta p \, \Delta q$ or for marginal changes

$$\Delta R = q\Delta p + p\Delta q$$

Elasticity and revenue



Elasticity and revenue

The relative change of the revenue induced by a change in prices equals $\dfrac{\Delta R}{\Delta p} = q + p\,\dfrac{\Delta q}{\Delta p}$.

This change is positive if $q + p\,\dfrac{\Delta q}{\Delta p} > 0$ or $-\dfrac{\Delta q}{\Delta p}\,\dfrac{p}{q} < 1$.

$\Rightarrow$ If $\varepsilon_p < 1$, the revenue will increase with an increasing price.

Elasticity and revenue

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This change is positive if $q + p\,\dfrac{\Delta q}{\Delta p} > 0$ or $-\dfrac{\Delta q}{\Delta p}\,\dfrac{p}{q} < 1$.

$\Rightarrow$ If $\varepsilon_p < 1$, the revenue will increase with an increasing price.

- $\varepsilon_p > 1$: elastic demand
- $\varepsilon_p < 1$: inelastic demand

Elasticity and revenue

- A producer has to evaluate the effect of a price change on the one hand and a quantity change on the other hand
- What is the effect of a price change on demanded quantity and how does it change the revenue?
- The answer is the calculation of the marginal revenue (MR)

From $\Delta R = q\Delta p + p\Delta q$ we receive

$$MR = \frac{\Delta R}{\Delta q} = p(q) + q\,\frac{\Delta p(q)}{\Delta q}$$

Elasticity and marginal revenue

Example

- We take (inverse) demand $p(q) = a - bq$
- the derivative with respect to q is $\frac{\Delta p}{\Delta q} = -b$
- the price elasticity of demand is

$$\varepsilon_p = (a - bq)/bq$$

- marginal revenue equals

$$MR = \frac{\Delta R}{\Delta q} = p(q) - bq = a - 2bq$$